

## CHAPTER XII

### TELEGRAPH CIRCUITS—(Continued)

#### 81. Differential Duplex Systems

Although the bridge polar duplex is still in use on many grounded telegraph circuits it is gradually being replaced by a similar device known as the "differential duplex". This has certain advantages that will appear as the discussion proceeds. Its major departure from the older type of set lies in the use of differential polar relays in both the sending and receiving circuits.

The principle of the differential relay depends upon winding a magnetic core with two equal but opposing windings so that if equal currents flow in the same direction through both windings, the magnetic field produced by one winding will be exactly neutralized by that set up by the other winding. Furthermore, by the use of a permanent magnet and a split magnetic circuit, such a relay may be polarized like the receiving relay of the bridge polar duplex set. The magnetic circuit of a typical relay of this type is illustrated in Figure 155.

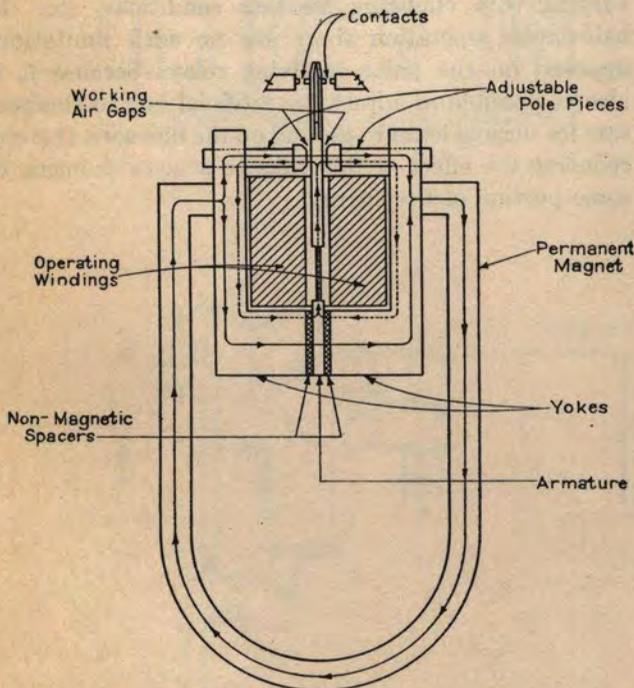


FIG. 155. MAGNETIC CIRCUIT OF THE DIFFERENTIAL POLAR RELAY

The manner in which the differential polar relay is employed to repeat signals in duplex telegraph operation may be best understood by referring to Figure 156,

which is a schematic drawing of a terminal differential duplex set arranged for full-duplex service. It will be observed that the familiar bridge arrangement of the bridge polar set is here replaced by a differential polar receiving relay, the two windings of which are connected at one end to the real and artificial lines respectively, and have their other ends connected together to the armature of the sending relay. When the artificial

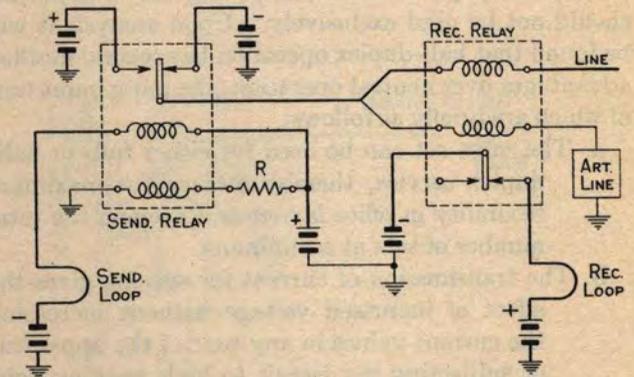


FIG. 156. TERMINAL DIFFERENTIAL DUPLEX SET ARRANGED FOR FULL DUPLEX SERVICE

line is adjusted to exactly balance the real line, currents coming from the sending battery divide equally between the two windings in parallel and, since these are connected differentially, the resultant magnetic flux is zero and the relay is not operated. Current coming from the line, on the other hand, flows through the two windings in series, which produces aiding magnetic fields and causes the relay armature to move to one or the other of its contacts depending upon the polarity of the incoming current. Thus it is evident that there is no interference between the sending and receiving circuits and the two can be operated quite independently of one another—in other words, full-duplex. It would be possible to operate this circuit with a neutral sending relay as in the bridge polar set but in practice a differential polar relay is used for this purpose also.

Referring again to Figure 156, the upper winding of the sending relay is known as the **operating winding** and the lower as the **biasing winding**. With the sending loop key closed, the magnetic fields produced by the two windings are in opposition because current is flowing in the same direction through each. The resistance,  $R$ , however, is of such value that the current

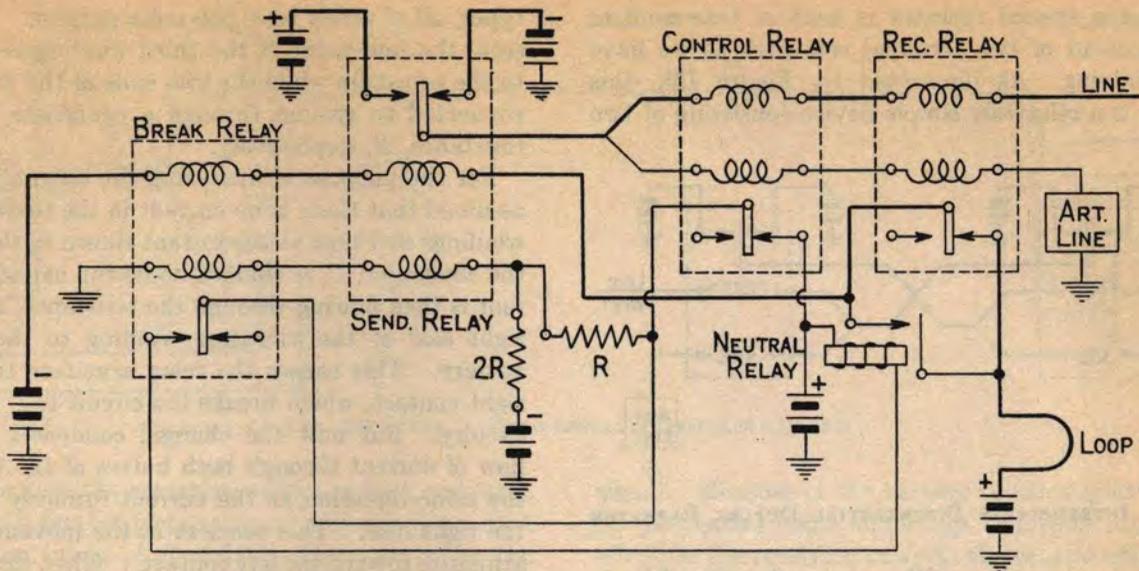


FIG. 157. TERMINAL DIFFERENTIAL DUPLEX SET ARRANGED FOR HALF DUPLEX SERVICE

in the biasing winding is limited to a substantially smaller value (usually half) than that in the operating winding and the armature is accordingly held to the marking contact. But when the sending loop key is opened, only the biasing winding is effective and the armature is drawn over to the spacing contact.

Half-duplex operation necessitates the inclusion of several additional relays in the differential duplex set circuit. The essentials of the circuit under these conditions are shown by Figure 157. The control relay, wired in series with the receiving relay, is provided to prevent the operation of the sending relay when the loop is opened and closed by the receiving relay. Two batteries of opposite polarity are connected in parallel to the biasing winding of the sending relay but the resistances,  $R$  and  $2R$ , are so adjusted in value that when the control relay contact is closed, the positive or spacing battery will be in control and the operation of the sending relay will be identical with its operation in the full-duplex circuit. When signals are being received, on the other hand, a spacing signal opens the loop at the contacts of the receiving relay and no current can flow through the operating winding of the sending relay. Then if it were not for the simultaneous operation of the control relay, the sending relay would be operated to spacing; but the opening of the control relay contacts breaks the circuit to the positive battery and allows current to flow in the opposite direction through the biasing winding and the resistance,  $2R$ , to the negative battery. This holds the sending relay armature on its marking contact.

An additional polar relay, known as the break relay, is connected in series with the sending relay. Its purpose will be understood if we analyze a condition where signals are being received from the distant

station and the local operator wishes to break. To do this he opens the loop circuit with his key. If at that instant a spacing signal is being received, this will have no effect because the loop circuit is already opened at the contacts of the receiving relay. But as soon as a marking signal is received and the control relay armature closes, spacing battery is connected to the biasing windings of the sending and break relays and since there is no current in their operating windings, both relays operate to spacing. The operation of the sending relay of course results in the transmission of the desired spacing or break signal to the line. The operation of the break relay insures that the break signal, once begun, will not be interrupted in case a spacing signal is received from the line. Such a received signal causes the control relay contacts to open and so would permit the sending relay to operate to marking were it not for the second connection between the spacing battery and the biasing windings of the sending and break relays, established through the spacing contacts of the latter relay. The neutral relay in series with this circuit is provided to take care of the possible contingency of the break and sending relays at both ends of a circuit becoming simultaneously operated to spacing. In such a case it would be impossible for either operator to regain control of the circuit because the loop circuits at both ends would be opened at the receiving relays. However, the operation of the neutral relay, which occurs whenever the control relay is opened **after** the break relay is operated to spacing, short-circuits the contacts of the receiving relay so that the sending and break relays will be operated to their marking contacts when the loop key is closed.

One of the important practical differences between bridge polar and differential duplex systems is that in

the latter a special repeater is used at intermediate points instead of two terminal sets such as we have been studying. As illustrated by Figure 158, this repeater is a relatively simple device consisting of two

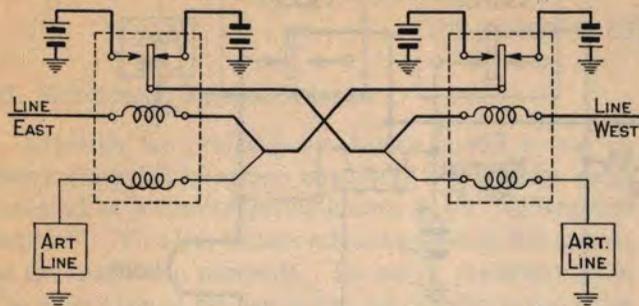


FIG. 158. INTERMEDIATE DIFFERENTIAL DUPLEX REPEATER

differential polar relays with associated artificial lines, by means of which signals are repeated directly from the Line East to the Line West and vice versa. Where a subscriber or a branch line is connected at an intermediate point, it is of course necessary to use terminal sets in much the same way as illustrated in Figure 154 for bridge polar duplex operation. Differential type intermediate repeaters can be used on circuits equipped at their terminals with bridge polar sets and bridge polar intermediate repeaters may be used on circuits equipped at one or both terminals with terminal differential sets.

From the foregoing it will be seen that the differential duplex system offers about the same general possibilities of operation as the bridge polar system. Several of its design features, however, make it more suitable for use on grounded lines than the bridge polar system, particularly where high-speed teletypewriter circuits are involved. The principal advantages are the use of a break relay operating simultaneously with the sending relay, which establishes the break circuit when the latter relay operates instead of after a sounder is released by the sending relay, and the employment of polar relays for transmitting as well as for receiving.

## 82. Principle of the Vibrating Circuit

Undoubtedly the most important advantage of the differential system lies in the rapid response to signals of the polar relays used. In addition to being inherently much more sensitive than the types of relays ordinarily used in bridge polar sets because of their more delicate design, these relays are equipped with a special third winding which forms a part of a "vibrating circuit" that adds further to their sensitivity and rapidity of response. The principle of the vibrating circuit may be understood by referring to Figure 159, which is a schematic drawing of one of several possible

types, all of which have the same purpose. As will be seen, the mid-point of the third winding is connected to the armature while the two ends of the winding are connected to ground through a condenser,  $C$ , and a resistance,  $R$ , respectively.

For the purpose of analyzing the circuit, it may be assumed that there is no current in the two main relay windings and that at the instant shown in the diagram the condenser,  $C$ , is charged to its full capacity. Current is then flowing through the resistance,  $R$ , and the right side of the vibrating winding to the negative battery. This causes the relay armature to leave its right contact, which breaks the circuit to the negative battery. But now the charged condenser sets up a flow of current through both halves of the winding in the same direction as the current formerly flowing in the right half. This accelerates the movement of the armature toward the left contact. When the armature reaches the left contact, there is a strong initial current from the positive battery through the left side of the winding to charge the condenser in the opposite direction. This current flows in the same direction in this

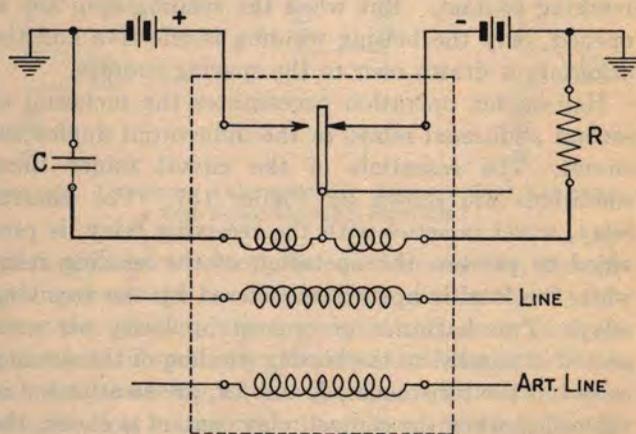


FIG. 159. VIBRATING CIRCUIT OF DIFFERENTIAL RELAY

half of the winding as the current flowing before the contact was made and its effect, therefore, is to hold the armature firmly against the contact without rebound or "chatter". As the condenser becomes charged, the current in the left side of the winding falls off until it becomes smaller than the current flowing in the opposite direction through the right side of the winding and the resistance. The armature then pulls away from the left contact; the current flowing from the condenser, which is now charged in the opposite direction, hastens its travel to the right contact, through which current then flows from the negative battery to again charge the condenser, thus holding the armature solidly against the contact. When the condenser is charged, the cycle is completed and it continues to repeat itself indefinitely.

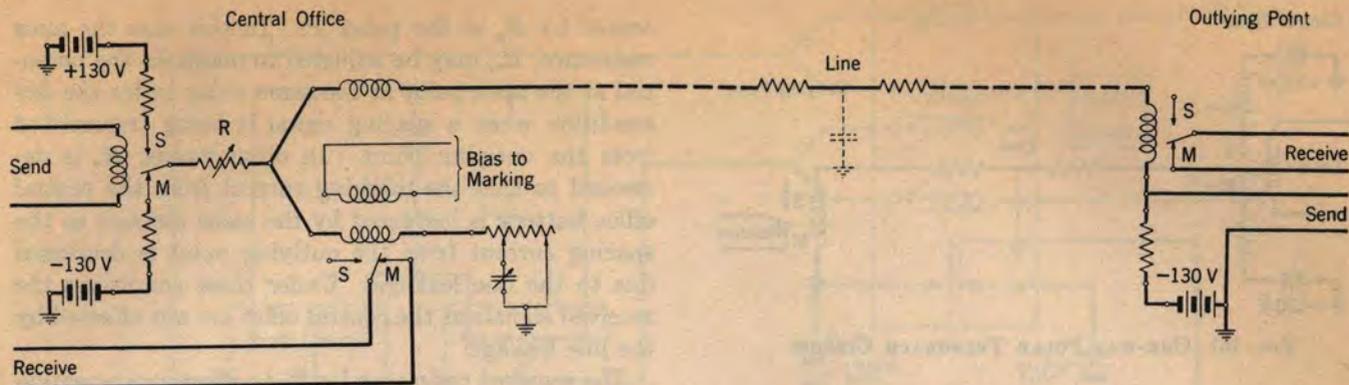


FIG. 160. TYPE A POLARENTIAL TELEGRAPH CIRCUIT

Thus we have the armature vibrating back and forth between the two contacts at a rate which is dependent only on the values of  $R$  and  $C$ . However, when the main windings of the relay are connected into the repeater circuit in normal fashion, the marking or spacing currents flowing in them will prevent the armature from vibrating freely under the influence of the vibrating circuit. But the tendency to vibrate is nevertheless present and whenever the current in a main winding is reversed due to the transmission of a signal, the vibrating circuit causes the armature to move from one contact to the other a little in advance of the time that it otherwise would. It also causes the movement to be more rapid and the contact to close more positively than would be the case if it were not operative.

### 83. Polarential and One-Way Polar Systems

For furnishing service to subscribers at outlying points, two special types of grounded telegraph systems known respectively as "Polarential" and "One-Way Polar" are frequently used. The polarential system permits true polar operation from the central office out and a modified polar operation from the outlying point into the central office. Thus the advantages of polar transmission are secured and at the same time the equipment arrangements in the subscriber's office are relatively simple. A schematic diagram of such a circuit is shown in Figure 160, with only the essential elements included for the sake of simplicity.

By inspection of the above diagram it may be seen that with the central office duplex repeater balanced while the outlying sending loop is closed, the transmission from the central office out is true polar. The relay at the outlying point receives signal combinations of equal marking and spacing currents of opposite polarity applied to the line by the sending relay at the central office.

In transmitting to the central office, ground is applied to the line at the outlying point for the marking

signal. Because of the balance of the duplex repeater at the central office under this condition, there is no effect on the receiving relay at the central office and it is held in the marking position by the current through the biasing winding. For the spacing signal, negative battery is applied to the line at the outlying point, which produces an effective spacing current in the receiving relay at the central office. This current comes from two sources, the first being due to the current flowing in the relay windings from the negative battery at the outlying point, and the second coming from the home battery as a result of the duplex unbalance caused by the resistance in series with the battery at the outlying point.

The variable resistance  $R$  at the central office is adjusted to such a value that the spacing line battery at the outlying point is higher than the potential applied to the apex of the repeater circuit at the central office. This assures that the line current will reverse when a spacing signal is transmitted from the outlying point. This is necessary in the case of teletypewriter operation to assure getting "home copy" at the outlying point.

Service conditions are sometimes such that only one-way transmission from the central office to an outlying point is required; as for example, in the transmission of news copy in certain cases. In such situations a somewhat simpler arrangement, known as the "One-Way Polar" system, is used. This also gives the advantage of polar operation but does not include the duplex feature at the central office. The essentials of this circuit arrangement are shown in Figure 161.

### 84. Leakage Compensation System

A somewhat novel method of operation employing polar transmission is shown schematically in Figure 162. The important advantage of this method of operation is that the circuit is self-compensating to a considerable extent for line leakage. This system is also sometimes called "Polarential (Type B)" in which

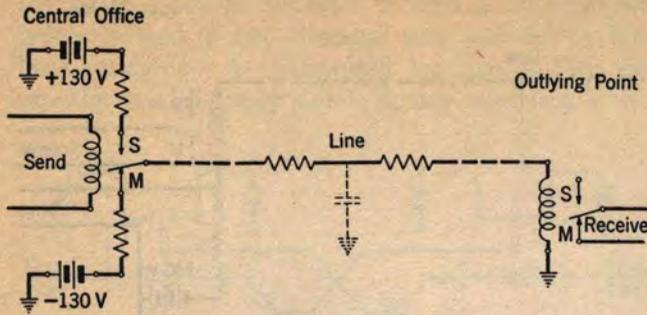


FIG. 161. ONE-WAY POLAR TELEGRAPH CIRCUIT

case the polar system discussed in the preceding article is designated "Polarential (Type A)".

It may be noted that true polar transmission is employed in sending from the central office to the outlying point. When sending at the central office, the receiving relay is held on its marking contact by the biasing current as shown. Transmission in this direction is therefore the same as in the polarential method of operation.

When transmitting from the outlying point to the central office on a *dry line*, the marking line current has no effect on the receiving relay at the central office due to the balancing network precisely balancing the line. For the spacing signal, aiding battery is applied to the line at the outlying point which causes an effective spacing current,  $\frac{E}{R_L}$ , to flow in the receiving relay, where  $E$  is the potential of the outlying battery and  $R_L$  is the resistance of the line (and the artificial line) from apex to ground. The biasing current is adjusted to a value of  $\frac{E}{2R_L}$ . With this set-up the signals sent at the outlying point will be satisfactorily received at the central office. When the line is *wet* there will be a comparatively large leakage to ground which may be repre-

sented by  $R_g$  at the point  $P$ . In this case the apex resistance,  $R_a$ , may be adjusted to maintain the potential at the apex point at the same value as for the dry condition when a spacing signal is being transmitted from the outlying point. In other words,  $R_a$  is decreased so that the marking current from the central office battery is increased by the same amount as the spacing current from the outlying point is decreased due to the line leakage. Under these conditions the received signals at the central office are not affected by the line leakage.

The required resistance for  $R_a$  to effect compensation for the line leakage may be determined from the following equation:

$$R_a = \frac{R_L(2R_2 - R_1)}{2R_1 + R_2} \quad (42)$$

where  $R_a$  and  $R_L$  are as previously defined,  $R_1$  is the resistance in the line from the apex to the point of leak, and  $R_2 = R_L - R_1$ .

From inspection of this equation, it is evident that if  $R_1$  is greater than  $2R_2$  complete leakage compensation cannot be effected unless the transmitting voltages at the central office are made higher than those at the outlying point. However, since  $R_2$  includes the resistance,  $R_b$ , in the outlying point, this latter resistance may readily be made great enough so that this condition will not ordinarily occur.

In sending a spacing signal from the outlying point, the batteries applied to the two ends of the line are aiding and the line is charged to a high positive potential near the outlying point. When the sending relay at the outlying point goes from space to mark, the high charge on the line causes a surge of current to flow to ground through the receiving relay in a direction opposite to that of the normal marking current. This would tend to cause the receiving relay at the outlying

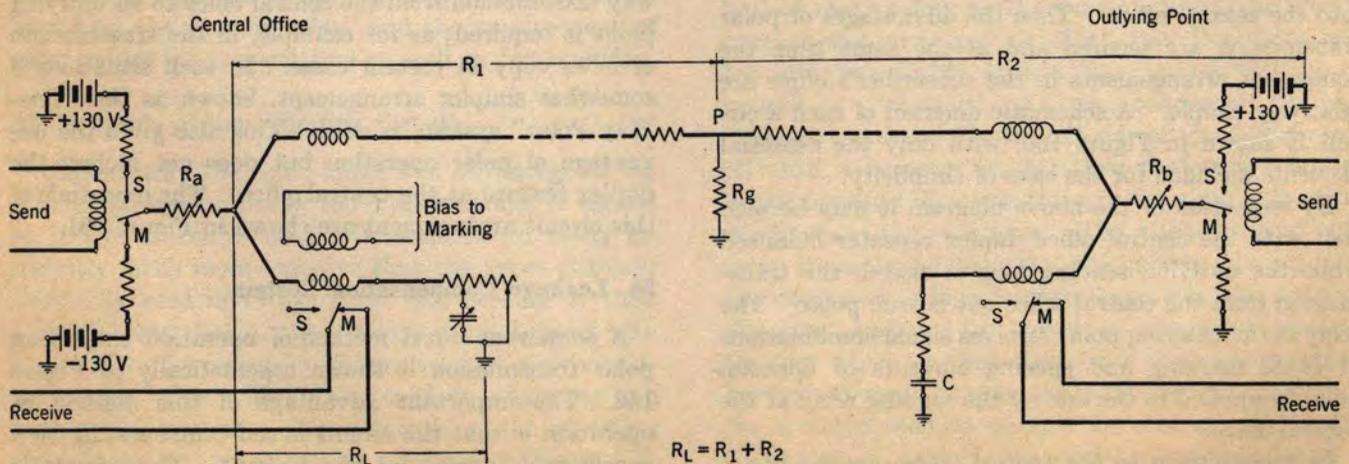


FIG. 162. TYPE B POLARENTIAL TELEGRAPH CIRCUIT

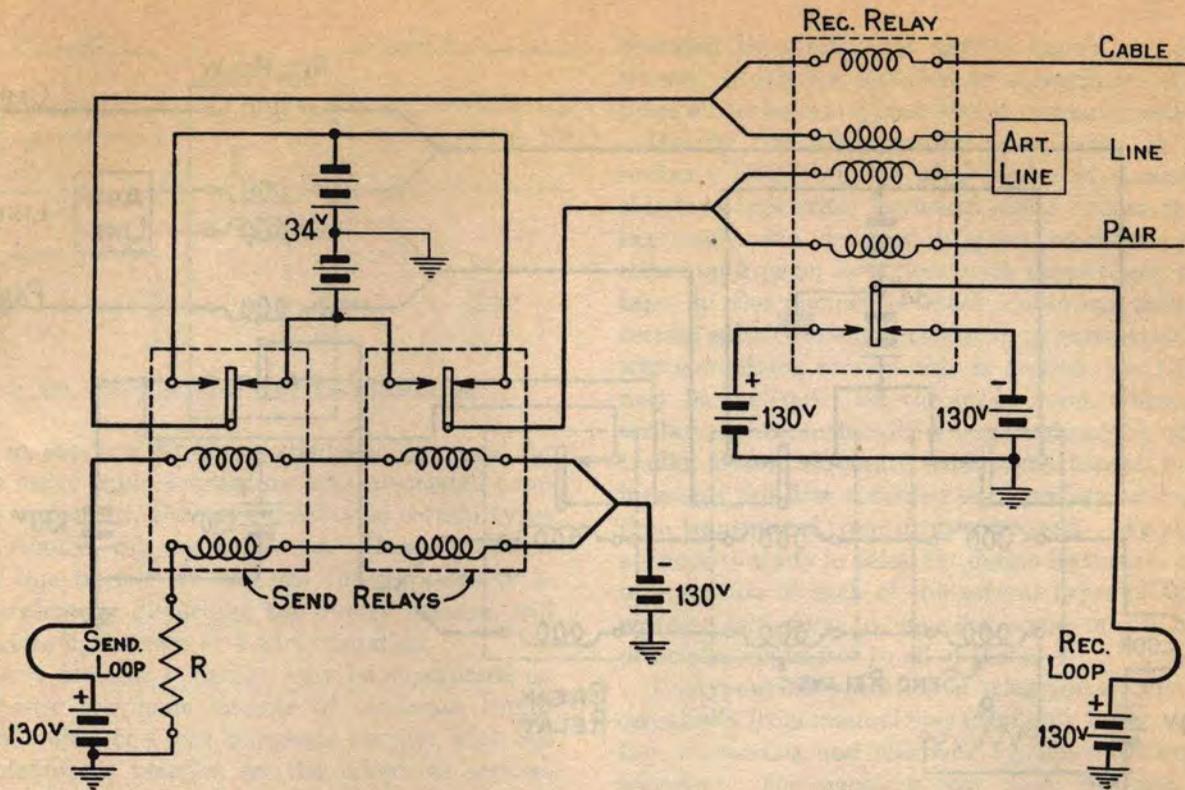


FIG. 163. TERMINAL METALLIC DUPLEX SET ARRANGED FOR FULL DUPLEX SERVICE

point to "kick off" and mutilate the home copy or produce false breaks. To neutralize the effect of this line "kick", the bridge arm containing the condenser *C* is added. This produces a "kick" affecting the relay oppositely to the line "kick" and of such magnitude that the receiving relay remains steadily on its marking contact when the sending relay is operating.

### 85. Metallic Telegraph Systems

The operation of telegraph circuits on composited cable conductors usually imposes certain requirements differing from open wire operation. In order to avoid interference with the telephone circuits, it is necessary in the first place that the telegraph currents be limited to values of the same order of magnitude as the telephone currents. Furthermore, in order to eliminate interference from ground potentials and crossfire and also from power circuits, it is preferable to use a second metallic conductor instead of an earth return, as is done in open wire operation. This means that at least two line wires are employed for each telegraph circuit. However, metallic telegraph systems are operated in practice both 2-wire and 4-wire.

The 2-wire metallic cable telegraph system is quite similar to the differential duplex system for grounded lines and, like the latter, employs different types of repeaters at through and terminal points. Figure 163

is a schematic drawing of a terminal metallic set arranged for full-duplex service, with the monitoring connections and all other auxiliary circuit details omitted for clearness. Here it will be noted that polar differential relays and balanced circuits are employed in both the sending and receiving circuits. The receiving relay has four balanced operating windings connected differentially so that when the artificial line is adjusted to balance the real line, there is no interference between incoming and outgoing signals. Polar transmission over the line is accomplished by means of a 34-volt battery which is reversed by the two sending relays to produce the marking and spacing signals. The line current, when the system is operating on 19-gage conductors, is approximately five milliamperes in each conductor of the pair. The sending loop is balanced by the resistance, *R*, and when the key is closed, the current in the upper or operating windings of the sending relays is exactly twice that in the lower or biasing windings because there are two batteries aiding in the loop circuit as against a single battery in the balancing circuit. Although the magnetic fields set up by the currents in the two sets of windings are in opposition, the preponderance of current in the operating windings holds the relay armatures on their marking contacts. When the loop key is opened, however, only the spacing windings are energized and the armatures are accordingly operated to spacing.

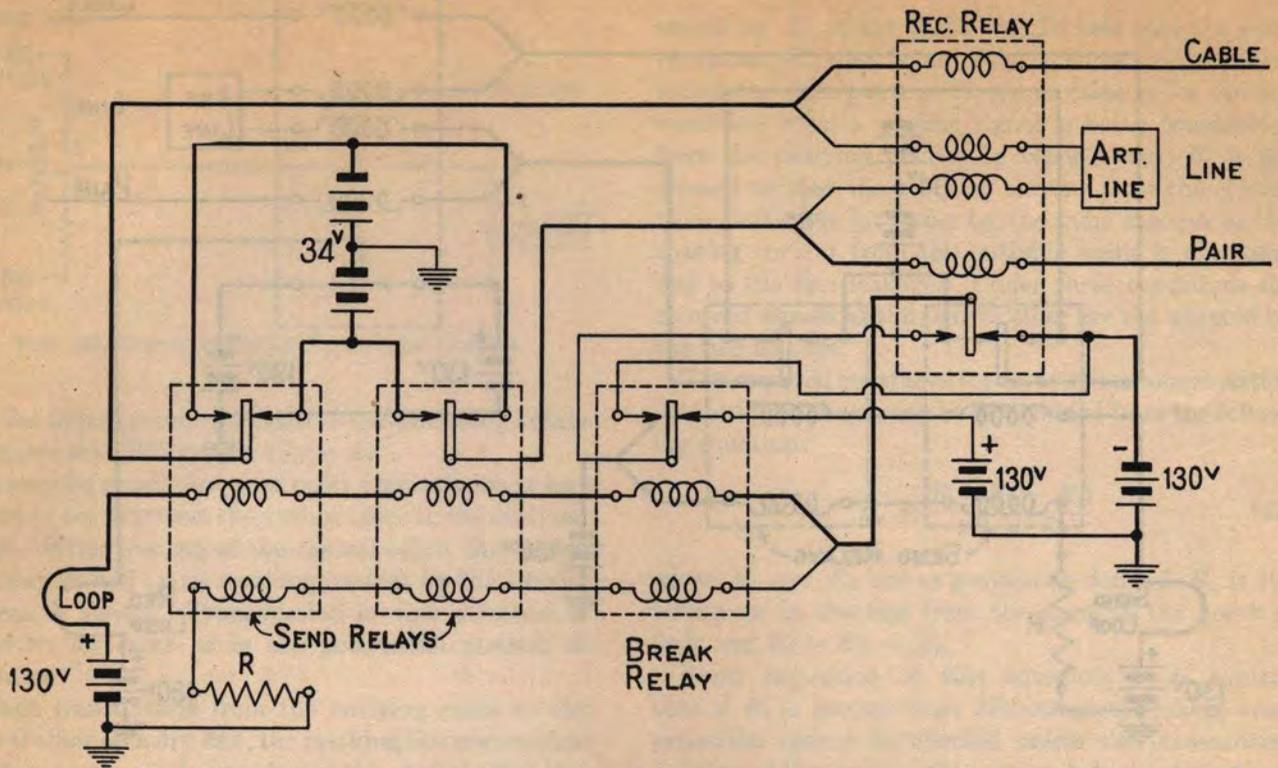


FIG. 164. TERMINAL METALLIC DUPLEX SET ARRANGED FOR HALF DUPLEX SERVICE

Half-duplex operation requires the addition of a break relay to the circuit, as shown in Figure 164. This permits the transmission of a clean-cut break by insuring that the armatures of the sending relays, once shifted to their spacing contacts by the opening of the key, remain so shifted. If the sending and receiving circuits were connected together to the loop without the break relay, opening the key in the loop would cause the sending relays to be controlled by the current flowing from the receiving relay contacts through their biasing windings and the balancing resistance to ground. Then if signals were being received at the time the key was opened for a break, the sending relays would be operated in accordance with the received signals, and these would be transmitted back over the line inverted, instead of the clean-cut spacing signal desired. But the break relay, connected in series with the sending relays as shown, operates simultaneously with the sending relays when the key is opened for a break. The shifting of its armature to the spacing contact connects negative battery to both contacts of the receiving relay so that the sending relay armatures are held on their spacing contacts as long as the loop key is open, regardless of the operation of the receiving relay by signals coming in from the line.

It is of course possible to use two terminal sets for through repeating at an intermediate point where no subscriber's loops are involved. In practice, however,

a special type of repeater is used for through operation which is much simpler in design. As shown schematically in Figure 165, this consists merely of two relays with artificial lines associated. In order to avoid the use of four relays instead of two, separate positive and negative 34-volt batteries are used. This is known as "single-commutation", as distinguished from the method of reversing the connections to a single battery in the manner described in connection with the terminal sets, which is known as "double-commutation". The polar relays in both terminal and through sets are equipped with vibrating windings which operate in substantially the same manner as described in Article 82.

In the 4-wire metallic telegraph system different paths are employed for transmission in the two directions, thus avoiding the necessity for networks or artificial lines to balance the line circuits. As indicated in Figure 166, which is a schematic of a 4-wire metallic telegraph circuit between two terminal type repeaters, the sending and receiving paths are separated from each other as regards transmission over the line. The local circuit arrangements of the repeaters are the same as for 2-wire operation.

Telegraph transmission with this 4-wire arrangement will, in general, be better than that obtained with 2-wire operation because of the improvement in stability produced by eliminating the duplex balance requirements. As the use of different paths for trans-

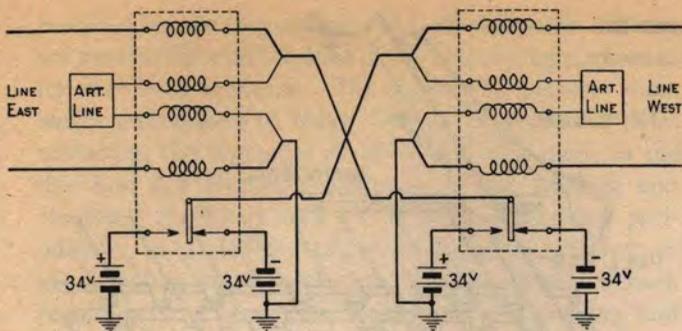


FIG. 165. INTERMEDIATE METALLIC REPEATER

mission in the two directions results in the need for twice as many cable conductors with associated compositing equipment, the susceptibility to certain types of line trouble will be increased. However, other types of line trouble which cause interruptions to 2-wire operation by disturbing the duplex balance, will not interfere in the case of 4-wire operation.

Four-wire metallic telegraph may be superposed on either 2-wire telephone circuits of moderate length (500-1000 miles) or 4-wire telephone circuits, with but little unfavorable reaction on the telephone service. However, very long 4-wire telephone circuits are not composited throughout their entire length because of low-frequency "delay distortion" introduced by the composite sets.

### 86. Principles of the Teletypewriter

In discussing telegraph circuits in the preceding articles, we have tacitly assumed that the signals to be transmitted were produced manually by the hand operation of an ordinary telegraph key. As a matter of fact, a large percentage of the telegraph circuits are

operated by mechanical devices known as teletype-writers, which are installed in subscribers' offices in place of the keys and sounders of manual practice.

Usually the teletypewriter installation at a subscriber's office consists of a keyboard similar to a standard typewriter keyboard and a typing or printing mechanism designed to print received messages either on a page, as is done with typewriters, or on a tape, in the manner of stock quotation tickers. At certain subscriber's stations, such as newspaper offices, where receiving service only is desired, the keyboard may be omitted. On the other hand, when a subscriber wishes to handle a large volume of outgoing traffic, a more elaborate sending mechanism in which messages are first recorded on a perforated tape and then transmitted from it, may be used. We shall not attempt to study in detail the design features or method of operation of each of the several types of teletype-writers, but only to consider some of the general principles applicable to all of them.

Teletypewriter operation of telegraph circuits differs essentially from manual operation only in the substitution of sending and receiving machines for keys and sounders. The signaling code used, however, is not the Morse code of manual operation but a special one in which each letter or signal is made up of five units or elements of equal length. As illustrated by Figure 167, this code provides for the letters of the alphabet, the numerals, and several miscellaneous symbols of common use, as well as for the special operations or "stunts" that the machines must perform, such as line feed, carriage return, and miscellaneous switching and signaling features. The machines must then be so designed that when a certain letter key is operated at the sending machine, the marking and spacing signals

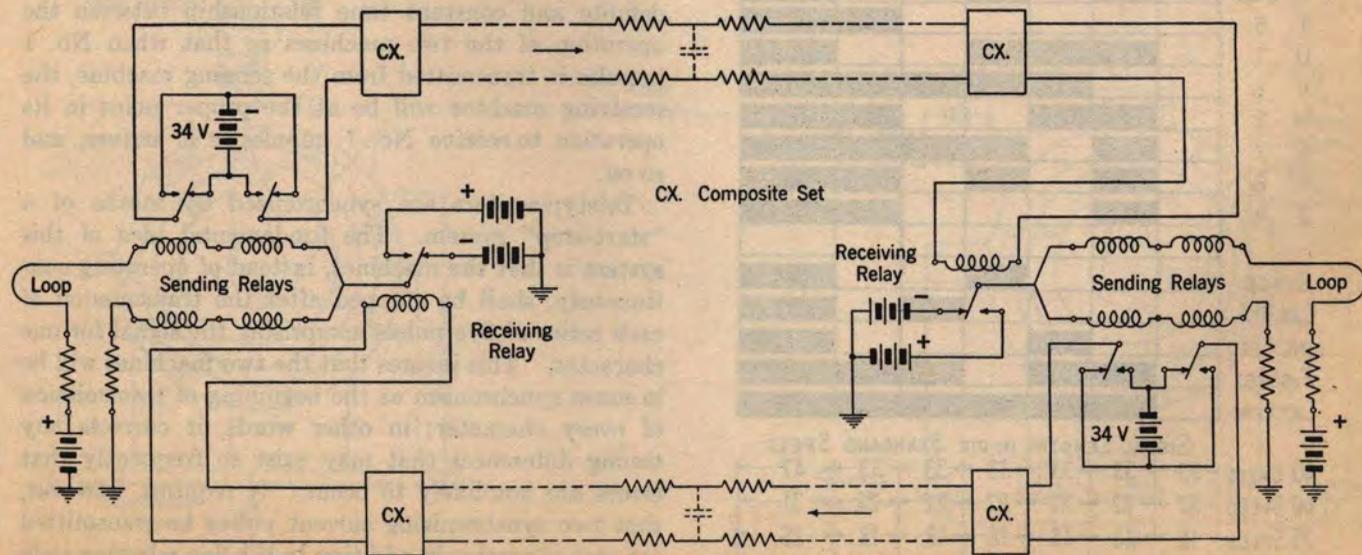


FIG. 166. 4-WIRE METALLIC TELEGRAPH CIRCUIT

corresponding to the code for that letter are sent out on the line; and when this signal combination comes in at the receiving machine, the corresponding type bar is selected and operated to print the letter.

The principle of selection may be understood by referring to Figure 168. The five "code bars" shown are under the control of the five signal units or pulses making up the code for each letter. If the first pulse is a marking signal, code bar No. 1 will be moved endwise a slight amount. Similarly code bar No. 2 will be moved or left in position accordingly as the second pulse is a marking or spacing signal, and so on through the five pulses of the code. When all five pulses have been received, the code bars are so ar-

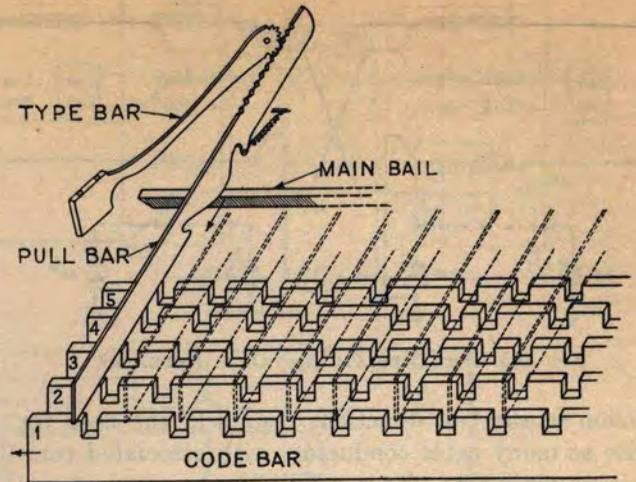


FIG. 168. SELECTING MECHANISM OF RECEIVING MACHINE

ranged that the slots under the "pull bar", corresponding to the particular code combination received, are in line and all other groups are out of line. This one pull bar is then allowed to drop down a small distance where it engages the "main bail" which pushes it forward and so causes the corresponding type bar to print the character.

Assuming some such selection method as has just been outlined and some analogous mechanical arrangement for producing the proper series of current pulses when a key is depressed at the sending machine, there remain two additional essential features that must be provided for. First, there must be a means of positioning the code bars in accordance with the incoming current pulses; this may be effected by electromagnets or by purely mechanical means. Second, and of vital importance, the sending and receiving machines must be kept in synchronism. That is to say, there must be a definite and constant time relationship between the operation of the two machines so that when No. 1 impulse is transmitted from the sending machine, the receiving machine will be at the proper point in its operation to receive No. 1 impulse as it arrives, and so on.

Teletypewriters are synchronized by means of a "start-stop" system. The fundamental idea of this system is that the machines, instead of operating continuously, shall be stopped after the transmission of each series of five pulses comprising the signal for one character. This insures that the two machines will be in exact synchronism at the beginning of transmission of every character; in other words, it corrects any timing differences that may exist so frequently that errors are not likely to occur. It requires, however, that two synchronizing current pulses be transmitted for each character in addition to the five selecting code pulses, a feature which necessarily extends the time

CHARACTERS	CODE SIGNALS						
	START	1	2	3	4	5	STOP
L.C. U.C.							
A -		█	█	█	█	█	
B ?		█		█	█	█	
C :		█	█		█	█	
D \$		█		█	█	█	
E 3		█	█	█		█	
F !		█		█	█	█	
G &		█	█		█	█	
H £		█		█	█	█	
I 8		█	█	█		█	
J ,		█		█	█	█	
K (		█	█		█	█	
L )		█		█	█	█	
M .		█	█	█		█	
N ;		█		█	█	█	
O 9		█	█	█		█	
P 0		█		█	█	█	
Q 1		█	█		█	█	
R 4		█		█	█	█	
S BELL		█	█	█		█	
T 5		█		█	█	█	
U 7		█	█	█		█	
V ;		█		█	█	█	
W 2		█	█		█	█	
X /		█		█	█	█	
Y 6		█	█	█		█	
Z "		█		█	█	█	
SPACE				█			
CAR.RET.				█			
LINE FEED				█			
FIGURES		█	█	█	█	█	
LETTERS		█	█	█	█	█	

	SIGNAL LENGTHS IN MS. STANDARD SPEED						
40 SPEED	33	33	33	33	33	33	47
60 SPEED	22	22	22	22	22	22	31
75 SPEED	18	18	18	18	18	18	25

FIG. 167. TELETYPEWRITER CODE

required for the transmission of each character. There are several different designs of the controlling apparatus for start-stop systems. The oldest and perhaps most easily understood of these consists of a pair of commutators, the segments of which are connected to the line and the electrical elements of the sending and receiving machines and are connected together periodically in a definite order by brushes rotating in synchronism and stopping at the completion of each revolution. A simplified diagram of the sending and receiving faces of a pair of these commutator devices, known as "distributors", is given in Figure 169.

To follow the operation, let us assume that the letter, D, is to be transmitted. By referring to Figure 167, we find that the code signal for this letter consists of a mark, two spaces, a mark, and a space. We must also remember that the start-stop system requires the transmission of two additional pulses, one to start the brushes revolving and one to complete the operation. The brush or "distributor arms" are coupled to the driving shafts of motors by friction clutches and are normally held stationary by the latches of the sending and receiving start magnets. The motors at the sending and receiving ends are governed to rotate at approximately the same speed. Now when a keyboard key (that for D in our example) is operated, the first effect is to close the circuit through the sending start magnet windings, which pulls up the latch and allows the sending distributor arm to start to rotate. As the inner pair of brushes passes over the start segment in the outer ring of the sending face, the line circuit is opened and a spacing signal is transmitted. This,

known as the "start-pulse", releases the receiving line relay, which connects battery to the receiving start magnet and permits the receiving distributor arm to start to rotate.

The operation of the key for D in the keyboard will also have connected battery to segments 1 and 4 of the sending distributor face in accordance with the code for that letter, so that when the sending distributor arm passes off from the start segment on to segment 1, the line will be closed to battery and the receiving line relay will operate. This will connect battery to the large inner segment of the receiving face with the result that when the receiving distributor arm passes over segment 1 (which it will do while the sending distributor arm is still on segment 1 of the sending face), selecting magnet No. 1 will be energized and will move its associated code bar in the printer mechanism. As the sending distributor arm passes over segments 2 and 3, "opens" will be transmitted to the line and accordingly no battery will be connected to the corresponding segments of the receiving face and the associated selecting magnets will not be operated. Continuing, selecting magnet 4 will operate while 5 will not. At this point the code bars in the receiving machine are properly placed for printing the letter, D, and as the receiving distributor arm passes off from segment 5 and on to the stop segment, battery is connected to the "printing magnet" which actuates the printing mechanism and causes the letter to be printed. In the meantime, the sending distributor arm has passed on to its stop segment, thereby transmitting a marking signal to the line, and is stopped by the start

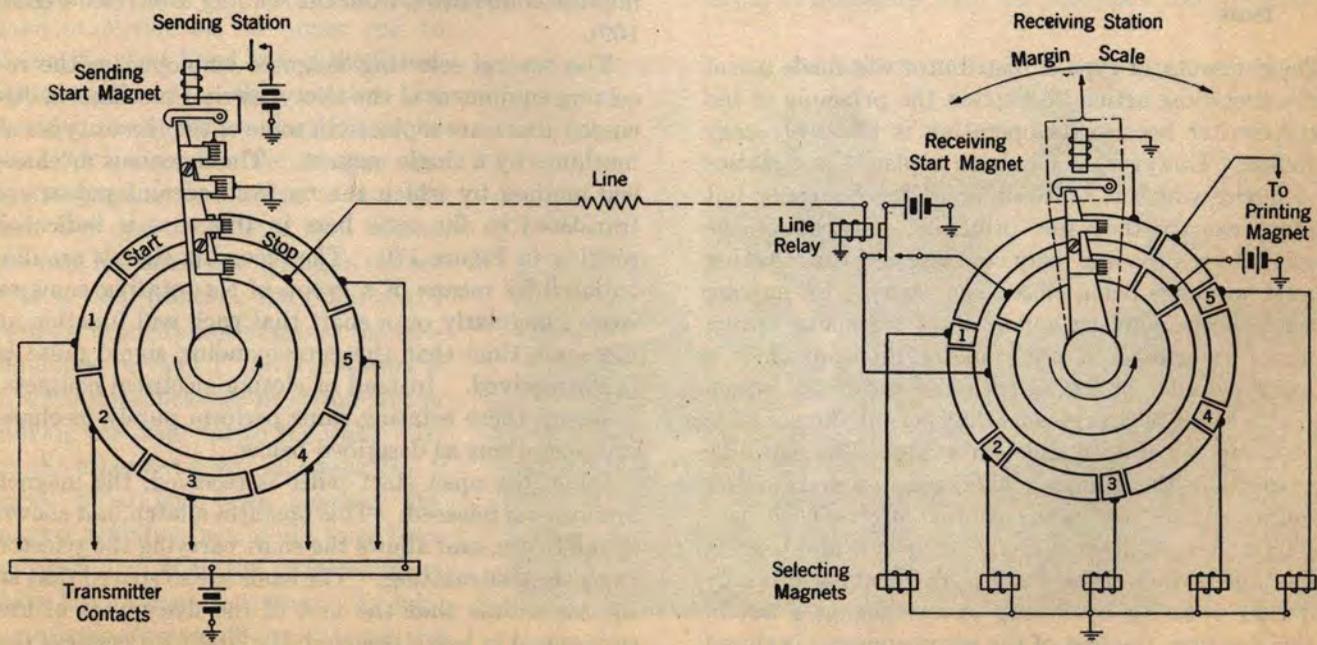


FIG. 169. THEORY OF START-STOP TELETYPEWRITER SYSTEM



No. 3A TWX SWITCHBOARD

magnet latch which was released as soon as the outer pair of brushes opened the circuit through the winding at the beginning of the operation. The received marking signal holds the receiving line relay closed so that the receiving start magnet is not operated and the receiving distributor arm is also stopped by its latch as it completes the revolution. Both distributors are then in position to handle the next character.

### 87. Operating Characteristics of Teletypewriter Systems

The commutator type of distributor was made use of in the preceding article to explain the principle of the teletypewriter because its operation is relatively easy to follow. However, it has been replaced in practice by a device which is quite different mechanically but employs exactly the same principle. Instead of the circuit between the line relay contacts and the selecting magnet windings being closed successively by moving brushes short-circuiting commutator segments, spring contacts are closed in order under the control of a rotating cylinder or drum, into the surface of which are cut a series of depressions that permit the contacts to close at the proper time intervals. The cam-like depressions in the drum are arranged in a spiral order about its surface and seven contact levers, each controlling a contact, are mounted side by side and bearing against the surface of the drum. The drum is normally held from rotating by a stop arm engaging a notch. In this position, the first of the seven contacts is closed because the first depression on the drum is then in

position to allow the contact lever to move forward. The circuit to the start magnet is connected through this closed contact. When a start pulse is received, the start magnet releases the drum and it starts to rotate, which immediately opens the first contact. Following this, the second depression on the drum comes under the second contact lever which then moves forward closing the contact to the first selecting magnet, and this magnet will be operated or not, depending upon whether the line relay is at that instant on its marking or its spacing contact. As the drum continues to rotate, the remaining four selecting contacts operate in order, which results in the remaining four selecting magnets being operated in accordance with the incoming signals; and finally, the seventh contact is closed to operate the printing magnet causing the character set up by the selecting magnets to be printed.

In order to increase the maximum overall speed of operation, the system may be so arranged that when sending at a maximum speed, the sending distributor rotates continuously instead of stopping after the transmission of each character. To preserve synchronism, however, it is still necessary that the receiving distributor come to a full stop after each complete revolution. This is effected by arranging the receiving distributor to rotate at some fourteen per cent greater speed than the sending distributor, thus providing a brief time interval during each revolution for it to stop. This requires, of course, that the depressions on the receiving cylinder be spaced fourteen per cent farther apart angularly than those on the sending drum in order that the receiving drum will close the receiving contacts during the exact middle portion of each signal impulse transmitted from the sending drum (see Figure 169).

The several selecting magnets employed in the receiving equipment of the teletypewriter mechanism discussed above are replaced in some of the recent types of machines by a single magnet. The ingenious mechanical method by which the received current pulses are translated to the code bars in this case is indicated roughly in Figure 170. The incoming signals are distributed by means of a group of six rotating cams so spaced angularly on a shaft that each will function at the same time that the corresponding signal pulse is being received. Instead of closing electrical contacts, however, these rotating cams perform purely mechanical operations as described below.

When the open start pulse is received, the magnet armature is released. This operates a latch, not shown in the figure, and allows the shaft carrying the selector cams to start rotating. The cams are so spaced that at the same time that the first of the five pulses of the code signal is being received, the first cam engages the projection on the "code bar operating lever" associated

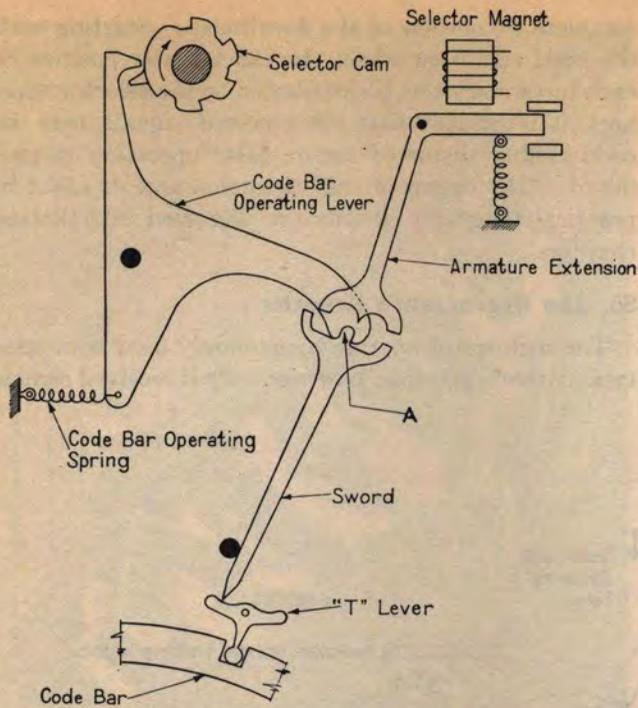


FIG. 170. RECEIVING MECHANISM OF SINGLE MAGNET TELETYPEWRITER

with the first code bar and rotates it slightly in a counterclockwise direction. The effect of this movement depends upon whether or not the magnet armature is operated. If the received No. 1 pulse is a marking signal, the armature will be operated as shown in the figure, whereas if this pulse is spacing, it will not be operated. But assuming that it is operated, the movement of the code bar operating lever by the first selector cam lifts up the "sword" and causes the right-hand projection on its upper end to strike the right-hand end of the "armature extension". This rotates the sword in a clockwise direction in its pivot "A", and when the selector cam in its continued rotation clears the code bar operating lever and allows the code bar operating spring to restore it to normal position, the point of the sword is brought down against the left-hand side of the "T" lever, rotating it in a counter-clockwise direction and so moving the code bar to the right. If, on the other hand, the incoming No. 1 signal pulse had been spacing, the magnet armature would not have been operated and when the code bar operating lever raised the sword, its left-hand projection would have struck the left-hand side of the armature exten-

sion causing the sword point to move to the right and the code bar to the left.

In exactly the same manner, when No. 2 pulse is received, the second selector cam will have arrived at the proper position in its rotation to operate the code bar operating lever associated with code bar No. 2 and it will be positioned according to the position of the magnet armature at the time. After all five signal pulses have been received and the code bars properly positioned, the sixth cam releases a clutch allowing the printing mechanism to operate.

One of the advantages of the teletypewriter over manual telegraph service is the high speed of operation that can be consistently maintained. There are three standard operating speeds, namely, 40, 60 and 75 words per minute. The lowest one of 240 operations or 40 words per minute corresponds to very fast manual operation. A speed of 360 operations or 60 words per minute is used for a majority of the teletypewriter services. The highest speed consists of 450 operations or 75 words per minute. Naturally these high signaling speeds require not only that the transmitting and receiving machines be sturdy and dependable but also that the connecting lines and telegraph repeating apparatus be of the highest grade.

Even though synchronism between sending and receiving machines may be satisfactorily maintained by means of the start-stop method of operation, it is clear that any distortion of the transmitted signals, due to unsatisfactory line conditions or other reasons, will tend to cause receiving errors. Therefore since a certain amount of distortion is practically inevitable in long telegraph circuits as we shall see in the next chapter, it is necessary that the teletypewriter systems be



PAGE TELETYPEWRITER WITH TAPE PERFORATOR AND AUTOMATIC SENDING EQUIPMENT

designed with the maximum possible operating margin. By referring to Figure 169, it will be noted that the segments on the receiving face of the distributor are considerably shorter than those on the sending face. This means that only a relatively small portion of each transmitted signal pulse is used for operation of the receiving machine and that these pulses may therefore vary appreciably before causing false operation of the receiving machine. This is illustrated by Figure 171 in which the narrow spaces bounded by the vertical lines indicate the time intervals during which the five selecting magnets are connected to the line during one

complete revolution of the distributor. Starting with the ideal condition where the exact center position of each incoming pulse is distributed to the selector magnets, it is evident that the received signals may be considerably distorted before false operation is produced. The causes of this distortion and its effect in practical telegraph circuits are discussed in following chapters.

### 88. The Regenerative Repeater

The high speed operation commonly used with typewriters is possible, however, only if received signals

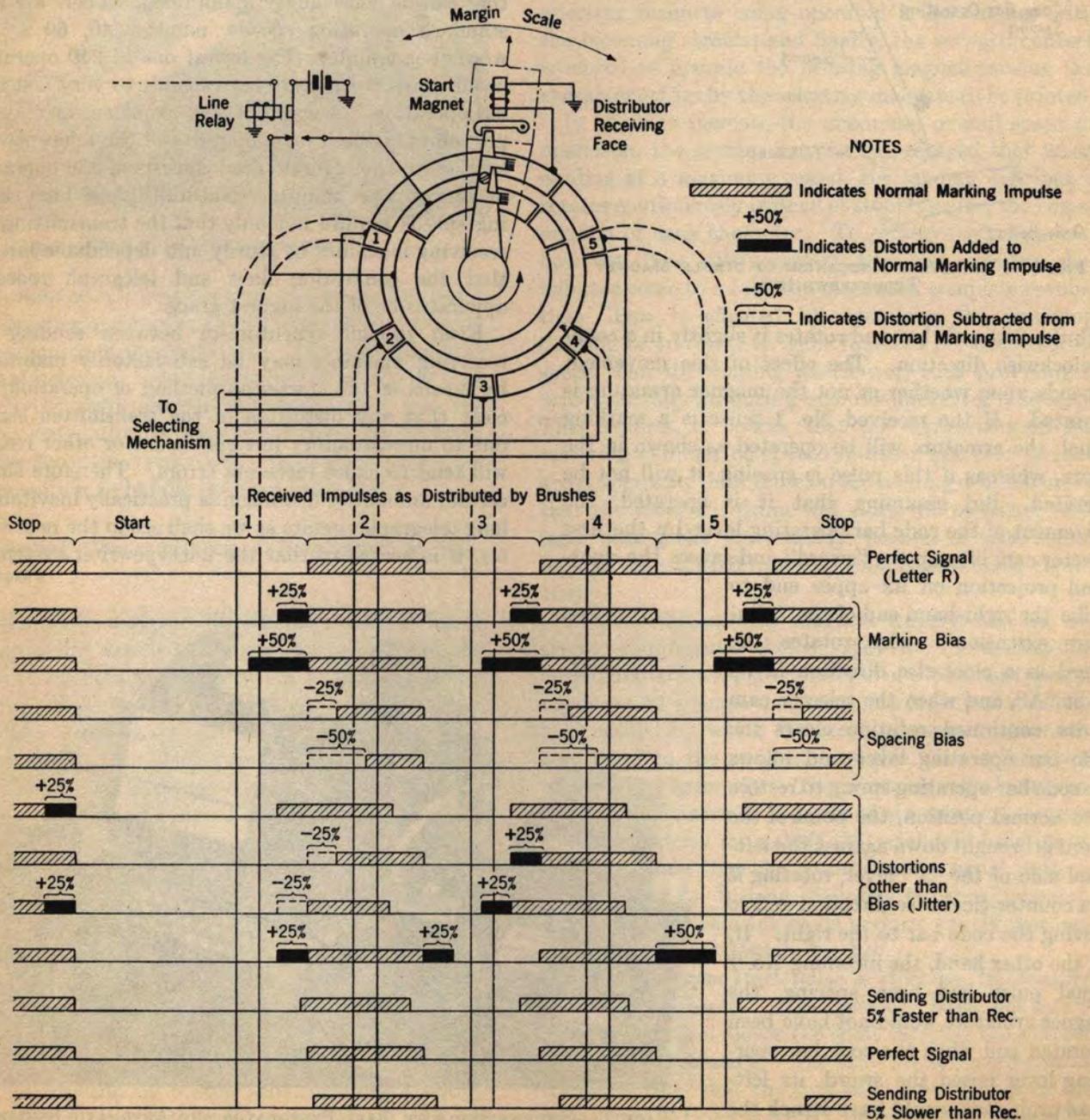


FIG. 171. TIME RELATIONSHIPS OF RECEIVED SIGNALS



LARGE TWX SWITCHBOARD

are free from any large amount of distortion. Since telegraph signals are invariably distorted to a greater or less extent in the process of transmission and since the ordinary telegraph repeaters repeat the greater part of such distortion so that it increases cumulatively with the length of the overall circuit, the maximum distance over which a teletypewriter circuit can be operated tends to be limited by this factor. Fortunately, the fact that the signals are of standard length and are transmitted with mechanical uniformity permits the use in long circuits of a special type of telegraph repeater which is capable of eliminating distortion from the signals.

This is known as the start-stop "regenerative" repeater. Consisting essentially of a sending and receiving drum or distributor-similar to those used in the teletypewriter mechanism, it is capable of receiving without error any set of signals that would be satisfactorily received by an ordinary teletypewriter set, and of sending these same signals out as free from distortion as the signals formed by the sending teletypewriter. Therefore by spacing regenerative repeaters at intervals that would be sufficiently short for satisfactory operation of a standard teletypewriter circuit, it is theoretically practicable to operate a circuit of any length whatever.

Figure 172 shows schematically the simplified ar-

angement of a type of regenerative repeater employing a flat distributor face and rotating brush arm. Here the two outer rings of segments represent the receiving commutator face and are shorted together by a pair of rotating brushes at the same time that the two inner rings, comprising the sending face, are shorted together by another pair of brushes mounted on the same rotating brush arm. To follow its operation, let us assume that the letter, R, is to be transmitted. Referring to Figure 167, we find that the incoming signals will consist of the starting spacing pulse, a space, a mark, a space, a mark, a space, and the final marking pulse. As the spacing start impulse is received, the brush arm will be released through a mechanism not shown in the drawing) and the two sets (of brushes will start to rotate. The receiving brush passes first over a blank segment and then connects the short No. 1 receiving segment to the receiving relay armature. This occurs at the instant that the first spacing signal of the five-impulse code is being received, and the receiving relay is therefore operated to its spacing contact. The right storing condenser will accordingly be charged positively. In the meantime, the brush of the sending face has moved over segment 7, connecting spacing battery to the sending relay and so repeating the start signal to the line in the other direction. Just after the receiving brush moves off from No. 1 segment on to a blank, the

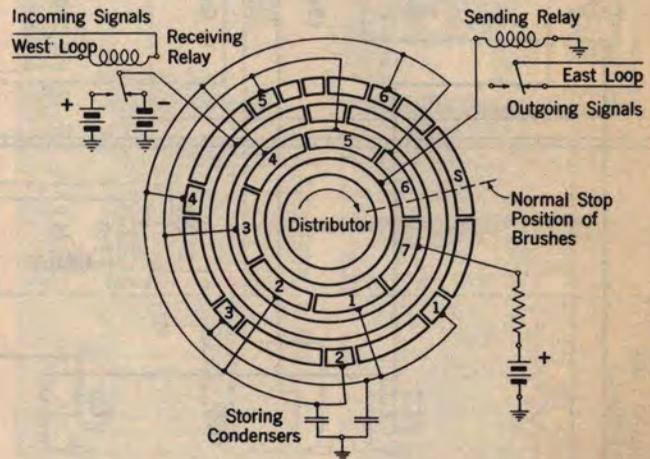
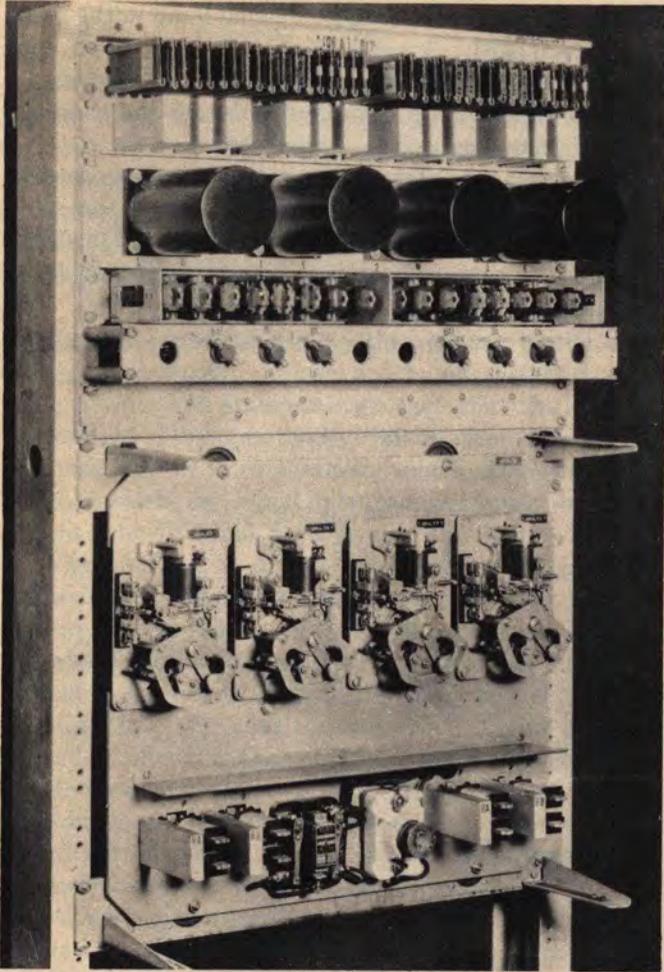


FIG. 172. PRINCIPLE OF THE REGENERATIVE REPEATER

sending brush moves on to No. 1 segment of the sending face and the positively charged right storing condenser discharges through it to the sending relay, thus repeating the first code spacing signal. During this operation, the receiving face brush moves on to segment No. 2 of the receiving face and charges the left storing condenser negatively in accordance with the incoming marking signal. This condenser is in turn discharged through No. 2 segment of the sending face



while the right condenser is being charged through No. 3 segment of the receiving face. This alternate operation continues until all five of the received code impulses and the final stopping mark impulse have been repeated to the outgoing line. As it completes its revolution, the brush arm is stopped until the next starting impulse is received.



CAM-TYPE REGENERATIVE TELEGRAPH REPEATERS

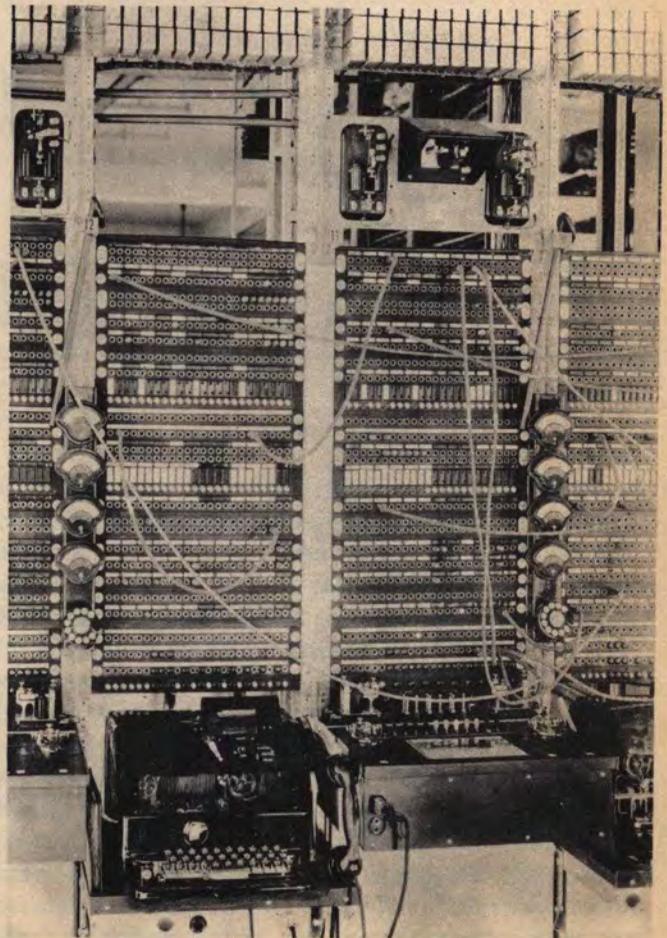
The fundamental value of the repeater lies in the fact that the short receiving segments pick up only the mid-portion of the incoming signal thereby allowing for considerable distortion, while the signals sent out are uniform and of equal length because the sending face segments are of equal length and spaced in exactly the same way as those of the regular sending distributor at the sending teletypewriter. It is apparent, accordingly, that the repeater will receive and convert to perfect signals any signals that are good enough to cause satisfactory operation of an ordinary receiving teletypewriter.

There is also a type of regenerative repeater which uses a cam type distributor. Its principle of operation is, however, the same as discussed above.

## 89. Typical Terminating Telegraph Circuit

In this and the preceding chapter we have studied the general principles of some of the more important equipment units that are used with long distance telegraph circuits, as well as the apparatus at the subscriber's station. In order that we may get a clear picture of how these several parts are coordinated in practice, Figure 173 shows in some detail the wiring arrangement of a representative terminating telegraph circuit from the subscriber's office to the long distance line wires. This includes in addition to the subscriber's station equipment and the terminal duplex set, the connections at the telegraph testboard and at the line testboard.

The repeater shown is a recent design of a terminal differential duplex set arranged for half-duplex operation and using a 2-wire line circuit. The usual ground return is employed for transmission and no signals are transmitted over the second line wire which is known as a "neutralizing wire". Its purpose is to balance out interference in the receiving relay that may be produced by the line section. This is accomplished by connecting the receiving relay windings to the regular



No. 9 TELEGRAPH TESTBOARD

line and to the neutralizing line so that they magnetically oppose each other. Any interference present on these line wires is then neutralized and has no effect on the receiving relay.

At the subscriber's station is a teletypewriter arranged for both sending and receiving. The telegraph testboard apparatus and wiring, which we have not hitherto considered, provide a flexible patching arrangement and also permit rapid testing of the telegraph facilities in both directions. The line circuits, coming from the regular telephone line testboard are connected direct to the differential duplex set, while the subscriber's loop is connected into "telegraph loop terminal" circuits (TLT) which are in turn connected to the duplex set.

The relays associated with the telegraph loop terminal circuit are a part of an auxiliary circuit by means of which a subscriber can signal an attendant at the

telegraph testboard. Normally the same current flows through the two opposing windings of the B-191 relay so that its armature is not attracted. But if the subscriber grounds one side of the loop by operating the push-button calling-in signal, the current in one winding of the B-191 relay becomes much larger than that in the other and the relay is operated. This connects battery to the E-206 relay winding through contacts of the Listening jacks to light a signal lamp in the telegraph testboard. An attendant at the telegraph line terminal answers the signal by plugging into the Listening jack which opens the circuit through the winding of the E-206 relay and the signal lamps.

Connections at intermediate repeater points on long telegraph circuits do not, of course, include the subscriber and telegraph line terminal circuits but in other respects the arrangement is generally similar to that shown by Figure 173.



## CHAPTER XIII

### TELEGRAPH TRANSMISSION PRINCIPLES

#### 90. Nature of Telegraph Signals

In telegraph transmission we are concerned with the reproduction of the sent telegraph message at the receiving end at a satisfactorily rapid rate, without error, and without interference to other services. Telegraph transmission differs from telephone transmission in that intelligence is conveyed from some sending point to one or more receiving points by means of a signal code. In the preceding chapters we assumed this was satisfactorily accomplished by the various circuits and apparatus discussed. However, the characteristics of these circuits and apparatus are such that the signals transmitted sometimes tend to fail to reproduce at the receiving end the same character that was transmitted. In other words, the signal in transmission may undergo certain changes which tend to alter its characteristics.

As pointed out in Article 76, there are two general methods of transmitting telegraph signals—(1) neutral transmission in which current is sent over the line to operate the relays to the marking position, and the current is stopped to operate the relays to the spacing position; and, (2) polar transmission which is accomplished by changing the polarity of the sending battery for the mark and space signals. Thus, telegraph signal transmission is accomplished on what may be termed a two current basis, that is, by transmitting spurts of steady current interspersed by intervals of no current in the case of neutral operation, or by transmitting spurts of current in one direction interspersed by reversals of current in the case of polar operation. In neutral operation, the closed circuit signal is referred to as a "mark" or "marking signal" and the open circuit signal is known as a "space" or "spacing signal". In polar operation, the marking and spacing nomenclature is retained but here it refers to the direction of current flow rather than to the open and close condition as in neutral operation. In either type of operation the change from one current condition to the other, that is, from mark to space or space to mark, is known as a **transition**.

The change of the current from the marking to the spacing condition, or vice versa, can be plotted with respect to time. A drawing showing this change of the current from the one condition to the other is called a "wave shape diagram" or, more commonly, simply a **wave shape**. Wave shapes, since they depict the

change of current in a telegraph circuit are an important aid in the study of telegraph transmission.

#### 91. Wave Shapes in Neutral Telegraph Systems

In any neutral telegraph circuit, if we could ignore the times required for the direct current to establish itself and to decay, the wave shape of a telegraph signal for the letter A in the Morse code would be as illustrated in Figure 174-A. As a practical matter, however, every telegraph circuit has some series inductance.

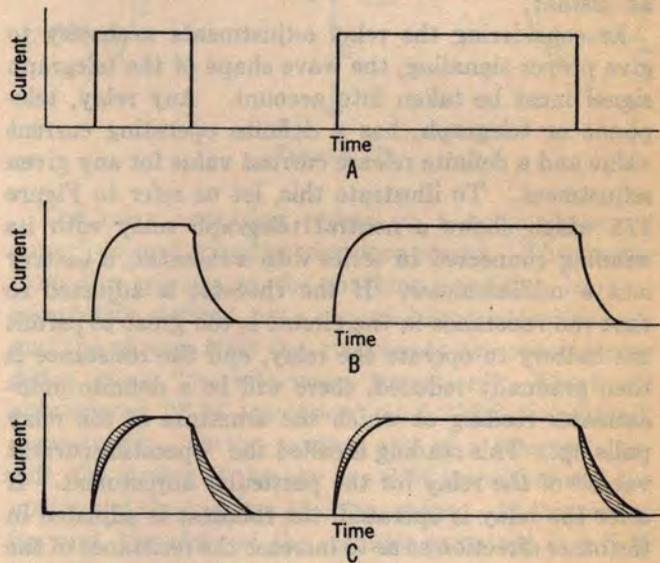


FIGURE 174

Each line relay adds some inductance and, in the case of the composited circuit, each retardation coil winding adds several henrys. The signal wave shape with series inductance is more nearly that represented by Figure 174-B, each current pulse having a sloping curve from zero to maximum value at the make of the key, and from maximum value to a point where the arc is broken at the break of the key. If in addition to the inductance we consider the condensers of the composite set, we have a further sloping of the pulse as shown by Figure 174-C. Here the shaded portion represents the effect of the condensers over and above the effect of the inductance. When the key is closed, the first rush of current flows only in part to the line; the inductance of the retardation coil in the composite set opposes any sudden change and diverts the current to the con-

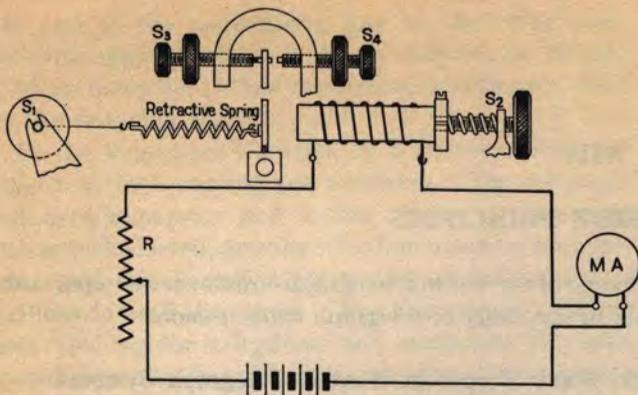


FIGURE 175

densers until they are charged to about the potential of the battery. When the key is opened, on the other hand, the current does not stop at the breaking of the arc because the discharging condensers sustain it for an instant.

In considering the relay adjustments necessary to give proper signaling, the wave shape of the telegraph signal must be taken into account. Any relay, telephone or telegraph, has a definite operating current value and a definite release current value for any given adjustment. To illustrate this, let us refer to Figure 175 which shows a neutral telegraph relay with its winding connected in series with a rheostat, a battery and a milliammeter. If the rheostat is adjusted so that the resistance in the circuit is too great to permit the battery to operate the relay, and the resistance is then gradually reduced, there will be a definite milliammeter reading at which the armature of the relay pulls up. This reading is called the "operating current value" of the relay for the particular adjustment. If after the relay is operated, the rheostat is adjusted in the other direction so as to increase the resistance of the circuit, the relay armature will fall back at a definite milliammeter reading. This is called the "release current" for the relay at the particular adjustment. The release current is smaller in value than the operating current for two reasons—(1) the magnetic circuit is much stronger when the armature is closer to the pole pieces so that the magnetic pull which holds the armature is greater than the pull which advances the armature; and (2) there is some residual magnetism in the iron core at the time the circuit is broken that did not exist at the time the circuit was made.

We might represent by the points *O* and *R* in Figure 176-A the operating and release current values respectively for a relay like that illustrated in Figure 175. With this particular adjustment, the length of the signal repeated by the relay will be the time indicated by *T*. If we should now make certain adjustments of the relay either by weakening the tension of the retractive spring

with the screw *S*<sub>1</sub>, lessening the air gap between the pole pieces and the armature with the screw *S*<sub>2</sub>, or decreasing the stroke of the armature by adjustments of the contact and back stop screws *S*<sub>3</sub> and *S*<sub>4</sub>, we may greatly decrease the operating and release current values, say to those represented by *O*<sub>1</sub> and *R*<sub>1</sub> of Figure 176-B. The effect would be to increase the length of the signal repeated by the relay from that represented by *T* to that represented by *T*<sub>1</sub>. These adjustments would have changed the signal from "light" to "heavy". For the sake of contrast, let us imagine that the wave shape of the signal was that shown by Figure 174-A. Here it is evident that we could neither increase nor decrease the length of the signal by relay adjustments.

To a degree this explains the frequent adjustments that are necessary on telegraph apparatus in practice. If additional inductance is added to a circuit by inserting a relay winding in series, the slope of the make and break of the signal is increased and a new adjustment may be required. The adjustment might be to lengthen the signal in one case and to shorten it in another. It would depend upon the original positions of points *O* and *R* on the curve.

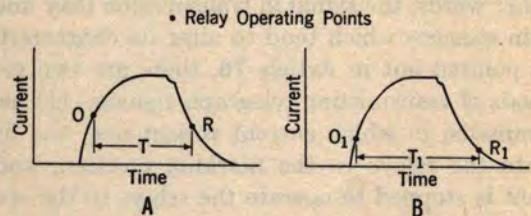


FIG. 176. EFFECT OF RELAY ADJUSTMENT ON TELEGRAPH SIGNAL LENGTHS

Another factor that will change the length of the signal is a change in the current value, resulting from a change in the voltage or in the series resistance. Let us consider the case of increasing the current by using higher voltage or taking series resistance out of the circuit. Naturally the operating and release current values of the relay before and after the change are the same, but they are more nearly the maximum current values before the change is made than after. Since the increase in current with constant inductance steepens the sides of the curve, the net result is an increase in the length of the signal. It is to be noted, however, that current values are limited in practice by considerations of crossfire and interference with telephone circuits, so that this is not ordinarily a practicable method of increasing signal length in actual operation.

In practice there is a third changing condition that affects adjustments. This is fluctuation in current values due to leakage along the line. To a degree it can be compensated for by using grounded battery connections at both ends of the circuit, one end being positive and the other end being negative, instead of

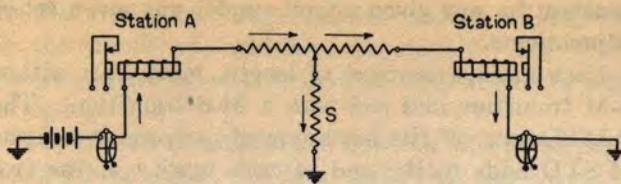


FIGURE 177

using a single battery at one end and only a ground at the other end. To understand this let us assume the condition shown in Figure 177 where  $S$  represents a leak to ground along the line, either distributed or otherwise. First, let us suppose that Figure 178-A represents the current curve when there is no leak. The leak will increase the current in the station A relay on account of the additional path through  $S$ . Since this path has less inductance we may represent the leakage current alone by the curve shown in Figure 178-B, which is smaller in value. Now, the curve  $B$  is going to influence the current through the A station relay by making it more nearly that represented by curve  $C$ , which is steeper and will therefore give a heavier signal. On the other hand, the leak  $S$  has a shunting effect on the current through the B station relay, and will not only tend to decrease the current but to flatten the curve as shown by Figure 178-D. This wide variation in the current through the two relays could not take place if the battery at Station A had about half its voltage, and an equal battery of opposite polarity was used at Station B, instead of a direct connection to ground. The current under these conditions will be more nearly constant through the relays at the two ends because we can assume that each battery is furnishing current to ground through the leak, and these currents as illustrated by Figure 179, tend to neutralize each other because they are flowing in opposite directions.

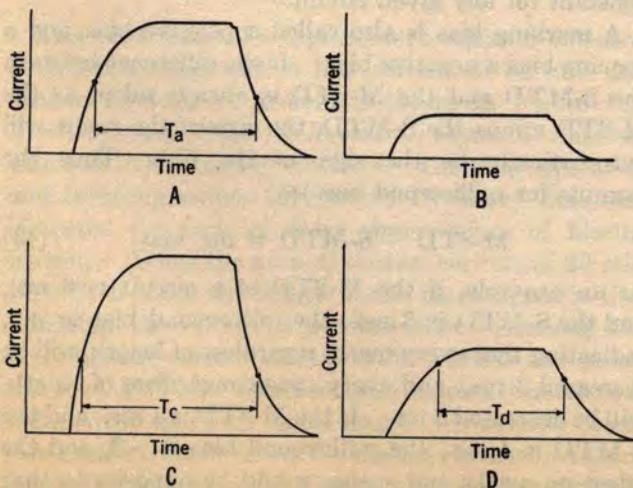


FIGURE 178

The shunting effect to ground of a leak, such as is shown by Figure 177, is a special case. On every telegraph wire, regardless of insulation conditions, we have in effect a leak to ground through the capacity between the wire and ground, or a condition that might be illustrated by substituting a condenser for the resistance  $S$  in Figure 177. Since the telegraph current wave shapes are somewhat similar to alternating-current cycles, the condenser may properly be considered as shunting the current to some extent. Furthermore, this capacity not only decreases the current value that reaches the distant station but tends to further distort the wave shape, thereby limiting the distance over which satisfactory signals can be sent without additional repeaters.

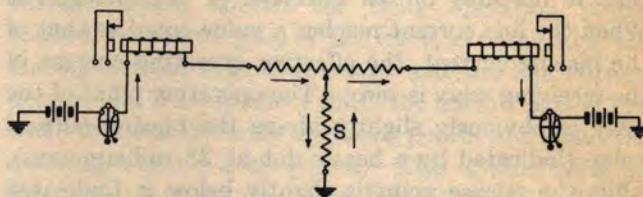


FIGURE 179

The relay operating points on a wave shape, of course, occur during the transition period. The change from the spacing to the marking condition is more completely defined as a "space-to-mark transition", and the change from the marking to the spacing condition as a "mark-to-space transition". These are abbreviated "S-M transition" and "M-S transition", respectively. At the sending end of a telegraph circuit, the closing of the key is a S-M transition and the opening of the key is a M-S transition. At the receiving end, the close of the sounder armature is a S-M transition while the release of the sounder armature is a M-S transition.

Let us consider the wave shapes of the telegraph signals in the neutral circuit, schematically illustrated in Figure 180, using an electrically biased receiving relay as discussed in Article 81. It will be seen from an inspection of this circuit that when the sending key is closed, the capacity between line and ground is charged by the current flowing from the battery at the sending end. The inductance in the circuit retards the current from building up to its full value instantaneously. The wave shape takes the form shown in Figure 181 (letter A in Morse code). As the biasing current in the receiving relay tends to hold the armature to the spacing contact, the line current, which magnetically opposes the biasing current, does not operate the receiving relay to the marking contact until it reaches a value slightly in excess of the biasing current. While the operating points of the relay are determined by its

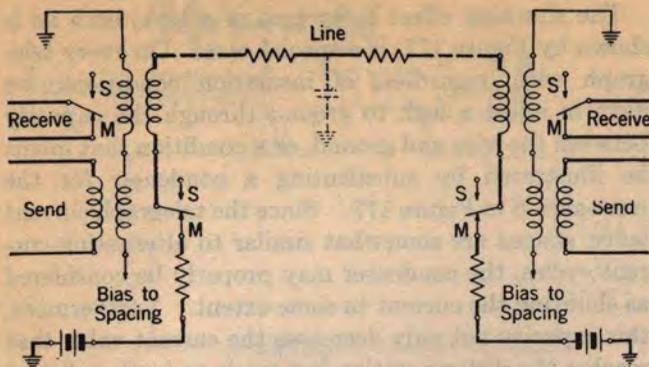


FIG. 180. NEUTRAL TELEGRAPH CIRCUIT EMPLOYING BIASED RELAYS

design and adjustments, we will consider for simplicity that it operates on an effective  $\pm 3$  milliamperes. When the line current reaches a value equal to that of the biasing current, the effective operating current in the receiving relay is zero. The operating point of the relay is obviously slightly above the biasing current value (indicated by a heavy dot at 33 milliamperes), while the release point is slightly below it (indicated by a heavy dot at 27 milliamperes).

At the instant the key at the sending end is closed, the line current starts to rise in the receiving relay as indicated by the wave shape but does not reach the relay operating point until a few milliseconds later. This means there is a delay between the closing of the sending key on a S-M transition and the operation of the receiving relay. This may be called a "space-to-mark transition delay" and abbreviated as S-MTD.

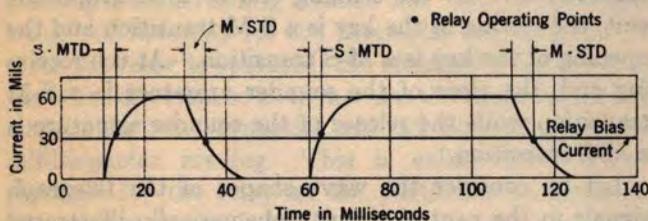


FIG. 181. SIGNAL WAVE SHAPES IN NEUTRAL TELEGRAPH CIRCUIT OF FIG. 180

In a similar manner when the sending key is opened, the line current in the receiving relay does not become zero instantaneously. The receiving relay will be held on its marking contact for an interval of time after the circuit is opened at the sending end. This time delay is a "mark-to-space transition delay" and is abbreviated as M-STD.

The magnitudes of these delays range from a fraction of a millisecond to several milliseconds. The S-MTD and M-STD are determined entirely by the characteristics of the circuit, and, though the two delays may not be equal, each transition delay will always be a

constant for any given circuit, under any given set of adjustments.

Each mark, regardless of length, must start with a S-M transition and end with a M-S transition. The S-MTD cuts off the beginning of each mark and the M-STD adds to the end of each mark. If the two delays are equal, the length of each mark will be unchanged by transmission over the circuit. Each space, regardless of length, starts with a M-S transition and ends with a S-M transition. The M-STD cuts off the beginning of each space and the S-MTD adds to the end of each space. Each delay thus has the opposite effect on a space that it has on a mark. If the two delays are equal, the length of each space will be unchanged by transmission over the circuit. The transmission is considered perfect if the received marks and spaces are exactly the same length as the sent marks and spaces.

## 92. Bias Distortion

The requirement for perfect transmission then is that the S-MTD equal the M-STD. If the two delays are not equal, as for instance if the M-STD is greater than the S-MTD, all marks will be lengthened, and all spaces will be shortened. This is a common condition on circuits and is called "marking bias" because the circuit lengthens the marks. If the S-MTD is greater than the M-STD, all spaces will be lengthened and all marks shortened. This is another common condition and is called "spacing bias".

Since the lengths of the marks and spaces may be indicated in milliseconds (ms.), the amount that is added to or subtracted from each mark or space due to a bias condition may also be indicated in milliseconds. It is equal to the difference between the S-MTD and the M-STD expressed in milliseconds. This is referred to as the "millisecond bias" of a circuit, and is a constant for any given circuit.

A marking bias is also called a positive bias, and a spacing bias a negative bias. If the difference between the S-MTD and the M-STD is always taken as the M-STD minus the S-MTD, the sign of the result will automatically be the sign of the bias. Thus the formula for millisecond bias is:

$$M\text{-STD} - S\text{-MTD} = \text{ms. bias} \quad (43)$$

As an example, if the M-STD of a circuit is 6 ms., and the S-MTD is 3 ms., the millisecond bias is +3, indicating that every mark, regardless of length, will be increased 3 ms., and every space, regardless of length, will be decreased 3 ms. If the M-STD is 1 ms., and the S-MTD is 4 ms., the millisecond bias is -3, and the effect on marks and spaces would be opposite to that in the first example. It is important to keep in mind

that a millisecond bias condition is determined entirely by the equipment, line facilities, overall length, etc. of the circuit and will be a constant for any given circuit, regardless of the speed of transmission or kind of signals.

The effect on transmission, however, of a given millisecond bias condition, does vary with the length of marks and spaces transmitted even though the millisecond bias condition itself is constant. As an example of this, let us consider a manual telegraph circuit where the dashes (long marks) are normally about two and a half to three times the length of the dots (short marks). In manual telegraph the lengths of the dots and dashes decrease as the speed of transmission increases. Assume first a slow speed of transmission where the dots are 30 ms. long and the dashes are 90 ms. long. A millisecond bias condition of +10 will make the dots 40 ms. long and the dashes 100 ms. long. The signals will be quite readable since the three to one ratio has been changed very little. Next, assume a much faster speed where the dots are 5 ms. long and the dashes 15 ms. long. The same +10 bias will make these dots 15 ms. long and the dashes 25 ms. long. Greater difficulty will be experienced in reading these signals since the dashes now are not even twice the length of the dots.

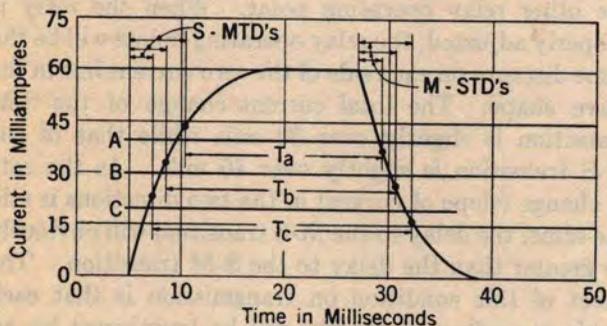


FIG. 182. EFFECT OF RELAY BIASING CURRENT ON SIGNAL LENGTH

The wave shape of a typical mark signal in a neutral circuit operating with a line current of 60 mils and having capacity to ground is shown in Figure 182. The horizontal lines A, B, and C, represent different values of relay biasing currents. The relay operating and releasing points (designated by heavy dots) are indicated for each of these three values of biasing current. When the normal biasing current of 30 mils (line B) is used, the length of the mark signal is that indicated by T<sub>b</sub>. It is obvious that increasing the relay biasing current increases the S-MTD and shortens the M-STD. This produces spacing bias since it reduces the marking signal length. A reverse condition results from lowering the biasing current to a value below the normal value; that is, the marking signal is

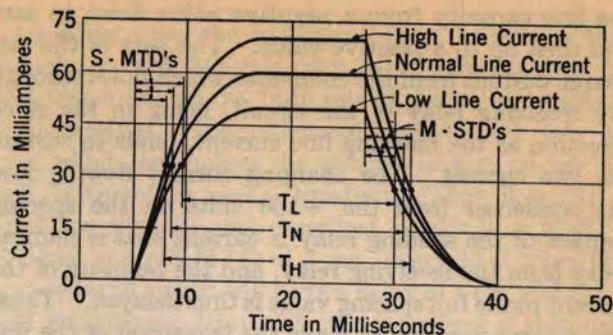


FIG. 183. EFFECT OF LINE CURRENT MAGNITUDE ON SIGNAL LENGTH

increased in length, as shown by T<sub>c</sub>, because the S-MTD decreases while the M-STD increases.

The same effect as that obtained by raising the bias current, which shifted the relay operating points toward the narrow part of the wave, is obtained if the biasing current is held constant and the line current decreased. This in effect shifts the narrow part of the wave towards the relay operating points, and again the marking impulse is shortened. This is illustrated by the wave shape for the "Low Line Current" in Figure 183. On the other hand, increasing the line current while the biasing current remains the same, increases the current at all points on the wave shape and effectively shifts the broader part of the wave toward the operating points. This lengthens the impulse as illustrated by the wave shape for the "High Line Current" in Figure 183. In other words, increasing the line current in a neutral circuit tends to produce marking bias and decreasing it tends to produce spacing bias.

### 93. Wave Shapes in Polar Telegraph Systems

A one-way polar circuit using a ground return is shown schematically in Figure 161. The sending relay connects -130 volts to the line for the marking condition and +130 volts for the spacing condition. The resistance at the sending end is adjusted by means of a potentiometer (not shown in the drawing) connected in the line circuit so that the current is normally about +35 mils for the marking condition and -35 mils for the spacing condition. These are, of course, the "steady state" values.

In this, as in other circuits, the change of the line current from marking to spacing (M-S transition) and from spacing to marking (S-M transition) will be delayed because of the capacity between the line and ground. When the line current is marking, the voltage on the condenser, representing the capacity between the line and ground, is negative. On the other hand, when the line current is spacing, the voltage on this condenser is positive. The change of the line current from marking to spacing then involves a change of the voltage on

the line capacity from a negative value down to zero and then up to a positive value. The part of the discharge current from the condenser which flows through the receiving relay of the circuit, being in the same direction as the marking line current, tends to sustain the line current. The charging current flowing into the condenser from the +130 volts on the spacing contact of the sending relay is current that is shunted away from the receiving relay, and the build-up of the current to the full spacing value is thus delayed. These two actions combine to make the transition of the line current from marking to spacing a gradual change which is represented by the polar wave shapes of Figure 184. The transition of the line current from spacing to marking may be analyzed in a similar manner to show the cause of the gradual change in this case.

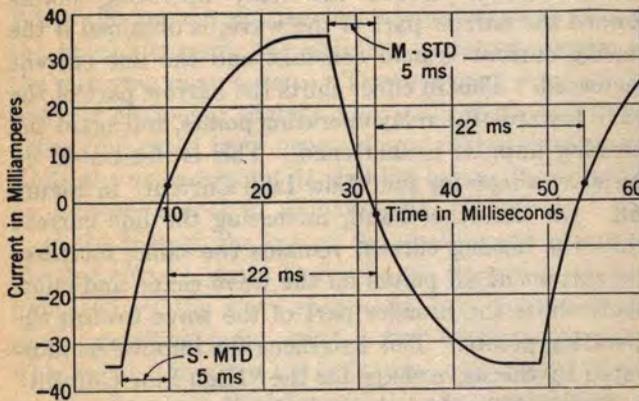


FIG. 184. SIGNAL WAVE SHAPES IN POLAR TELEGRAPH CIRCUITS

The fact that the M-S and S-M wave shapes are identical in form is a valuable feature of polar operation. To obtain the full advantage of this feature, however, the relay operating points must be symmetrically located on the wave shape. That is, the S-M relay operating point must be located the same distance from the start of the S-M wave shape, as the M-S relay operating point is located from the start of the M-S wave shape. These relay operating points will then be the same distance on each side of the zero current line of the wave shape diagrams. The S-MTD and M-STD are equal and there is no bias in the received signals.

Unbiased polar transmission thus depends upon three conditions—(1) that equal but opposite potentials be applied at the sending end; (2) that the resistance of the circuit remain constant for both positions of the sending relay armature, and (3) that the operating points of the relay be located symmetrically about the middle of the wave shape in order that equal transition delays will be secured.

Figure 185 shows a case where the steady state mark-

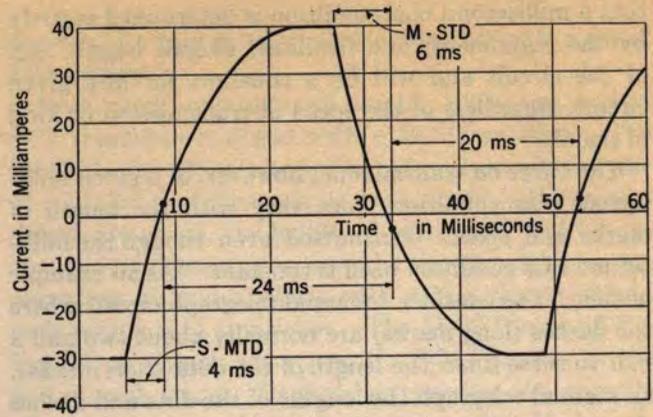


FIG. 185. EFFECT ON SIGNAL LENGTHS OF UNEQUAL POLAR LINE CURRENTS

ing and spacing currents of a polar circuit are not equal, the marking current being +40 mils and the spacing current being -30 mils. This condition might be due to a difference in ground potential between the terminals or to an unbalance between the voltages on the contacts of the sending relay. In this case a S-M transition starts when the line current is at -30 mils and ends when the current reaches the relay operating point, while the M-S transition starts when the line current is +40 mils and ends when the current reaches the other relay operating point. When the relay is properly adjusted, the relay operating points will be the same distance on each side of the zero current line in the wave shape. The total current change of the S-M transition is slightly over 30 mils while that of the M-S transition is slightly over 40 mils. As the rate of change (slope of curves) in the two directions is still the same, the delay to the M-S transition will obviously be greater than the delay to the S-M transition. The effect of this condition on transmission is that each mark, regardless of length, will be lengthened by an amount equal to the difference between the two transition delays, and each space, regardless of length, will be shortened by the same amount.

If the bias condition of the circuit were reversed, which would be the case if the spacing current were greater than the marking current, the delay to the S-M transitions would then be greater than the delay to the M-S transitions. Under this condition all marks would be shortened and all spaces would be lengthened and a spacing bias would exist.

A situation similar to the one just described would have existed if the steady state current values had remained normal and the relay operating points had been shifted one way or the other on the wave shape. This could be caused by a biased adjustment of the relay which, if it were marking would cause the relay to operate to marking more easily than usual, and would thus shift the S-M operating point down on the wave

shape. By the same token the relay would operate to spacing less readily, thus requiring more spacing current to operate it, and shifting the M-S operating point down on the wave shape also. This shifting of the operating points would once again make the transition from the marking condition to the M-S operating point on the wave shape different from the transition from the spacing condition to the S-M operating point on the wave shape. Unequal transmission delays and bias to transmission would result, just as in the previous case.

In either case, the important thing to note is that though the M-S transition delays are different than the S-M transition delays, both sets of delays are constant in themselves. The difference between the two delays, which determines the amount of bias on the circuit, is therefore also a constant. Thus if a circuit condition like the one described results in a M-STD of 5 ms. and a S-MTD of 3 ms., every M-S transition sent over the circuit will have a delay of 5 ms. and every S-M transition a delay of 3 ms., regardless of the interval of time that may exist between transitions.

## CHAPTER XIV

### TELEGRAPH TRANSMISSION PRINCIPLES—(Continued)

#### 94. Characteristic Distortion

In the discussion so far, a transition has been always assumed to start when the line current was at the steady state (full value) marking or spacing condition. There are situations, however, where the start of the transition does not occur when the line current is at its steady state value. As we know, a definite amount of time is required for the line current to change from the steady state marking condition to the steady state spacing condition, and vice versa. Thus in Figure 184 the time required for the current to make the complete change from marking to spacing and from spacing to marking is approximately 18 ms. On each transition in this case, the line current would have plenty of time to reach the steady state value before the next transition occurred. The following transition would then start from the same current value as the preceding transitions and the transition delay would be the same as the previous delays.

In actual practice, the time required for the current to change from one steady state condition to the other is sometimes greater than the minimum time interval between transitions in the signals. Some transitions then must occur while the line current is still in the process of changing from the previous transition. These transitions have a different delay time from transitions starting when the line current is in the steady state condition and must therefore be distinguished from the latter type.

Figure 186-A illustrates a case where the line current requires 33 ms. to change from the steady state spacing condition to the steady state marking condition. Now assume that a marking impulse 22 ms. long is being transmitted. The S-M transition at the start of the marking impulse occurs when the line current is in the steady state spacing condition of  $-35$  mils. This transition is thus a steady state current transition, and as such will have the normal S-M transition delay, which is the same for all steady state S-M transitions.

The S-M transition at the beginning of the marking impulse starts the current changing towards the steady state marking current value, an action which in this particular circuit will require 33 ms. to complete. However, the M-S transition at the end of the marking impulse occurs only 22 ms. later. At this time the line current, in the process of changing from  $-35$  mils to  $+35$  mils, has reached a value of  $+25$  mils. The

operation of the sending relay at the end of the marking impulse reverses the voltage applied to the line, and the line current accordingly ceases changing towards the marking condition, and starts back towards the steady state spacing condition. Since this M-S transition occurs when the line current is still in the process of changing, it is called a "changing current transition". When the line current reaches the value of  $-3$  mils, the receiving relay operates to spacing, completing the M-S transition on the circuit.

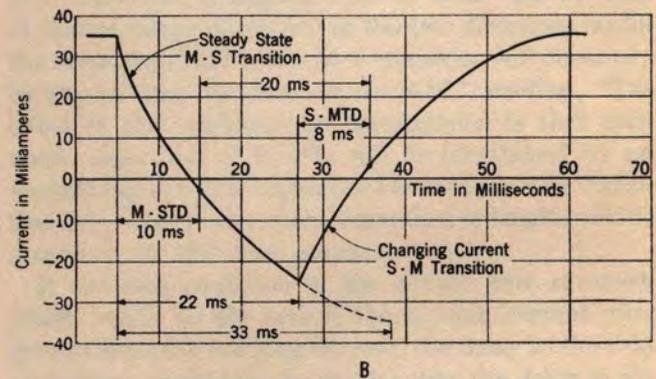
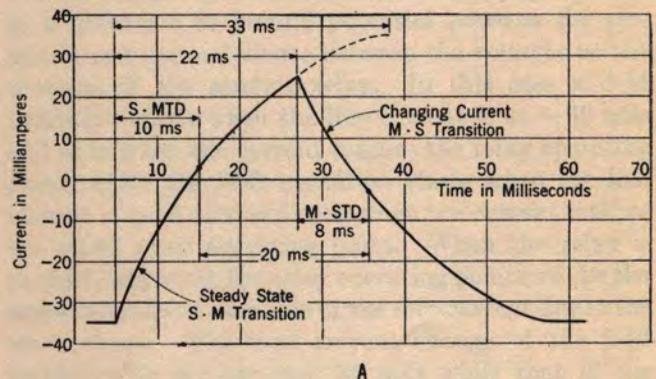


FIGURE 186

The net effect on the marking impulse being transmitted in Figure 186-A will be to shorten it 2 ms., since the transition delay at the end of the impulse (8 ms.), which adds to the impulse, is 2 ms. less than the transition delay at the start of the impulse (10 ms.), which subtracts from the beginning of the impulse.

In a polar circuit, the rate of change of the current from spacing to marking is, of course, the same as the rate of change from marking to spacing. Accordingly, since in this particular circuit 33 ms. were re-

quired for the current to change from  $-35$  to  $+35$  mils, 33 ms. will also be required for the current to change from  $+35$  mils to  $-35$  mils. It also follows, then, that if a spacing impulse only 22 ms. long is transmitted, the S-M transition at the end of the impulse will occur when the current is still in the changing condition and this transition will be a changing current transition.

Since, in the case of the current changing from spacing to marking, the value of the current at the end of 22 ms. was  $+25$  mils, it follows that in this case the current at the end of 22 ms. will be  $-25$  mils. This condition is illustrated in Figure 186-B: The total current change involved in the S-M transition at the end of the spacing impulse will then be from  $-25$  to  $+3$  mils or 28 mils, the same as the total current change that took place in the former case. Likewise, the delay to this changing current transition will be 8 ms. and the marking impulse being transmitted will then be reduced 2 ms. in length.

The magnitude of the changing current transition delays just discussed is proportional to the time required for the current to change from its value at the start of the transition to the operating point value of the receiving relay. In both Figures 186-A and -B the current change was from 25 mils to 3 mils of the opposite sign, or a total change of 28 mils. It is obvious, however, from an inspection of these figures, that if the impulse transmitted had been longer than 22 ms., the line current would have been at a higher value at the time of the transition at the end of the impulse, and the transition delay would have been greater. The limiting delay will, of course, be the steady state delay.

Also if the impulse transmitted had been less than 22 ms. in length, the line current would have been at a lower value at the time of the transition at the end of the impulse, and the transition delay would accordingly have been less. This is illustrated by Figure 187 which shows wave shapes of marking impulses for the three standard teletypewriter speeds in a circuit where the time required for the line current to change from its negative to positive value, and vice versa, is 33 ms. The marking impulses illustrated are 18 ms. long, corresponding to 75 speed operation; 22 ms. long corresponding to 60 speed operation; and 33 ms. long corresponding to 40 speed operation. Wave shapes for the spacing signals would, of course, be identical except for reversal of the current values.

In the case of the 33 ms. marking impulse, the impulse is just the required length for the current to change from one steady state condition to the other, and the transition at the end of the impulse is thus a steady state transition. In the case of the 22 ms. impulse the S-M transition at the end occurs when the line current is at  $+25$  mil value, and this transition is thus a changing current transition starting at a current

value less than the steady state value. Accordingly as we noted before, the delay is less than the delay to the steady state current transition, 8 ms. as compared to 10 ms. In the case of the 18 ms. impulse, the S-M transition occurs when the line current is only at  $+18$  mil value. This transition is thus also a changing current transition. Due to the fact that the line current only changes 21 mils to reach the M-S operating point of the relay, as compared to the change of 28 mils for the M-S transition of the 22 ms. impulse, the delay is still less. As indicated in the figure, it is now only 7 ms.

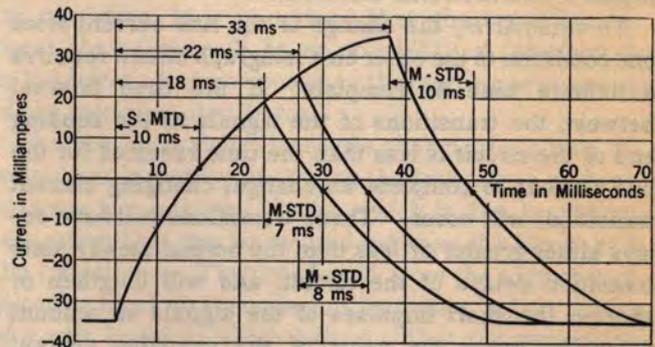


FIG. 187. CHARACTERISTIC DISTORTION EFFECTS ON SIGNAL LENGTHS AT 40, 60, AND 75 SPEED OPERATION

The amount of a changing current transition delay is thus dependent upon the value of the line current at the start of the transition. The value of the line current is dependent upon the time interval between the changing current transition under discussion and the previous transition, which started the line current to changing. Since the time interval between the beginning of these two transitions is equal to the length of the sent impulse, it is this impulse length which finally determines the transition delay under a given set of conditions.

In the condition just described, the lengths of the received signal impulses are obviously affected by the presence of the changing current transitions. This effect is called **characteristic distortion**. The magnitude of the effect is inversely proportional to the length of the sent impulses, and the nature of the effect is to shorten received **short** impulses. Since the received impulses under consideration are shortened, the effect in this case is called **negative characteristic distortion**. An opposite effect is possible. The characteristics of a circuit may be such that the line current tends to increase momentarily at the completion of each transition to a value greater than the steady state, due to transient effects. If the next transition occurs at such an instant, the transition delay will be greater than the delay on the preceding transition which means

the length of the received mark or space signal, as the case may be, will be lengthened. Since the signal impulse is lengthened, this is called **positive characteristic distortion**. However, as the transient effect causes the line current to oscillate (increase and decrease) around the steady state, it is possible that the next transition might occur at the instant the line current had momentarily decreased below the steady state. In such a case the transition delay would be less which would result in negative characteristic distortion. Thus, transient conditions may cause either positive or negative characteristic distortion, but positive characteristic distortion is not so frequently encountered as negative characteristic distortion.

To summarize, the change of the line current from one condition to the other on a telegraph circuit requires a definite time to complete. If the time interval between the transitions of the signals at the sending end of the circuit is less than the time required for the line current to complete its change, changing current transitions will occur. These transitions will have delays either greater or less than the normal steady state transition delays of the circuit, and will lengthen or shorten the short impulses of the signals an amount depending upon the value of the changing current transition delay, which in turn, is dependent upon the length of the impulse that caused the changing current transition. If the effect is to shorten the short impulses it is negative characteristic distortion. If the effect is to lengthen the short impulse it is positive characteristic distortion.

The contrasts between characteristic distortion and bias are as follows:

1. The effect of characteristic distortion depends upon the length of the impulses transmitted. The effect of bias is independent of the length of the impulses.
2. For a given length of impulse, the effect of characteristic distortion is independent of whether it is a marking or spacing impulse. The effect of bias is always opposite on a mark to what it is on a space.
3. Characteristic distortion is related to the amount and arrangement of the capacity, inductance and resistance of a circuit. Except in neutral operation, these factors do not effect bias.
4. Bias is caused by unequal marking and spacing line current, biased relays, etc., conditions which do not effect characteristic distortion.
5. Characteristic distortion, because it is due to the capacity, inductance and resistance of a circuit, which, except for the resistance, are unchanging in value, varies only a small amount from day to day on a circuit. Bias, because it is caused by unbalanced voltages, ground potential, re-

lays losing adjustment, etc., may vary from hour to hour on a circuit.

## 95. Fortuitous Distortion

The form of distortion, caused by such factors as crossfire, power induction, momentary battery fluctuations, "hits", break key operation and the like, and which displaces miscellaneous received transitions by various amounts intermittently, is known as **fortuitous distortion**. At times this effect may be large enough to produce a complete failure of the circuit. In the transmission of miscellaneous signals, the combined effect of characteristic and fortuitous distortion on the displacement of received transitions is sometimes known as "jitter".

## 96. Teletypewriter Margin Measurements

From the preceding discussion, it is apparent there is a need for some means of measuring the quality of telegraph signals as transmitted over various types of circuits and under varying conditions. Within certain limits, the teletypewriter itself may be used as a measuring instrument for this purpose.

As illustrated by Figure 169, only five successive equal signal intervals are required to provide combinations for all the characters normally used. These are supplemented by one equal open interval immediately preceding the group of five, for starting the rotation of the receiving distributor cam or brush arm; and, a closed interval immediately following, for the purpose of stopping the rotation of the receiving distributor after the group of five intervals have operated the selecting mechanism of the receiving machine. This closed stop interval is made equal to 1.42 times the length of each of the other six equal intervals. This longer interval insures that, under any condition normally encountered, the receiving distributor will be stopped before the next character combination is received. Using the start and selecting intervals as units, the sending distributor is so constructed that six open or closed intervals of one unit each, and one stop interval of 1.42 units are consecutively produced for each character transmitted.

The principal teletypewriter operating speeds used are 40 and 60 words per minute, with the latter predominating. The average word is assumed to consist of five letters and a space and it therefore requires six revolutions of the distributor brush arm for transmittal. At 60 speed, there are 60 times 6 or 360 revolutions per minute of the brush arm. However, as the brush arm is stopped and started once every revolution, the distributor driving shaft is operated slightly above this speed, i.e., approximately 368 instead of 360 revolutions per minute.

In one complete revolution, which requires 60 Sec./360 Rev. or .163 second per revolution (163 milliseconds), the brush arm passes over 7.42 units. Therefore when operating at 60 speed, the time for each unit signal impulse is approximately 22 milliseconds and the time for the long stop impulse is 31 milliseconds.

Figure 188-A indicates the sequence of circuit conditions produced by the sending distributor in transmitting the letter "R". Here the circular distributor is laid out as a straight line. The shaded areas represent the intervals during which the circuit is closed and the blank sections the intervals during which the circuit is opened by the sending distributor.

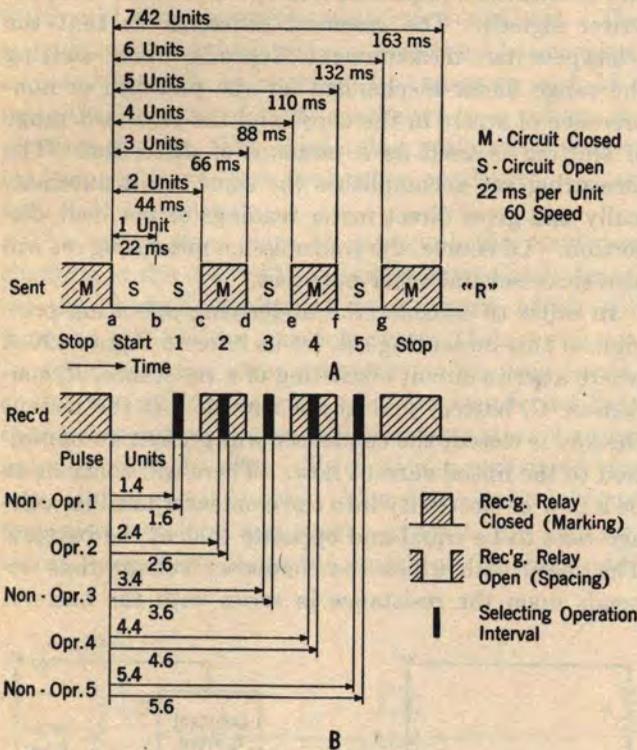


FIGURE 188

The received signals shown in Figure 188-B have the same time lengths as those produced by the sending distributor of Figure 188-A. The solid blocks superimposed upon the received signals represent those parts of the signals which are used by the selecting mechanism of the receiving machine (see Figure 169).

The selecting mechanism, when "oriented" correctly, is so arranged that it normally operates only during the central portion of the received signal impulse and requires only about twenty per cent of the unit interval. On this basis, the selection for pulse number 1 occurs during the period of time 1.4 to 1.6 units after the beginning of the received start interval, the selection for pulse number 2 occurs 2.4 to 2.6 units after the start,

and the remaining pulse selections occur in a similar manner 3.4 to 3.6, 4.4 to 4.6, and 5.4 to 5.6 units, measured in each case from the beginning of the received start interval.

For the transmission of the letter "R" as shown in Figure 188-A, there are mark-to-space transitions at points *a*, *d* and *f*, and space-to-mark transitions at points *c*, *e* and *g*. For some other character combination, a transition may occur at point *b*, but in any transmitted character there can be only two, four or six transitions.

Inasmuch as the selecting functions take place only during the intervals shown by the solid blocks of Figure 188-B, and require twenty per cent of each unit interval for operation, it is important that the transitions so occur that there will be no possibility of interference to the selecting operations or to the starting or stopping of the receiving distributor.

For the ideal signal intervals shown, the above requirement is met by producing the transitions midway between the selecting blocks, which is the maximum separation that can be secured between the blocks and the transitions. The time length from the edge of each selecting block to the adjacent transition is four tenths of a unit interval, which indicates that the transitions may be shifted towards the selecting blocks as much as forty per cent of the length of a unit interval before an error is recorded on the machine. Any deviation from the ideal positions for the occurrence of transitions represents distortion and may be measured in terms of its percentage of a unit interval. Thus the above machine is able to tolerate a maximum distortion of about forty per cent.

As we have seen, distortion in the form of bias or characteristic distortion displaces the signal transitions so as to effectively shift the position of the received mark or space impulses in some definitely systematic way.

The ideal situation, of course, is for the selecting segments of the receiving distributor, which are one-fifth the length of the unit segments in the sending distributor, to be at mid-position with respect to the sending units. Under these conditions the transitions may be shifted as much as forty per cent in either direction before an error is recorded by the machine. The receiving unit of the teletypewriter machine is equipped with a mechanism whereby the latch assembly (or distributor face) may be mechanically moved through an arc corresponding to the length of a unit segment. By this means all of the selecting segments may be shifted with respect to the beginning of the start segment (receiving brush arm released) over a scale range equal to a unit segment (22 milliseconds for 60 speed). This mechanism is known as a "range finder" and is equipped with a scale graduated from 0

to 120 as indicated in Figure 189. One hundred divisions on this scale represent an arc equal to a unit segment. This arrangement provides a means of measuring the distortion on received signals.

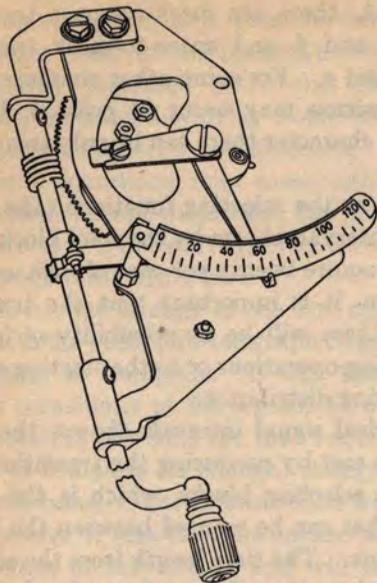


FIG. 189. TELETYPEWRITER RANGE FINDER MECHANISM

To measure the total net effect of all kinds of systematic distortion, or the position of received signals, the range finder is first moved in one direction until errors appear in the "copy" and then moved back slowly until these errors are just eliminated. Similarly, the range finder is moved the maximum distance before errors occur in the opposite direction. These two scale readings then give the operating margin of the signals under test. On perfect signals the margin would be from 10 to 90, since the effective received signal must occupy twenty per cent of the total range.

Margin measurements, in addition to showing the distortion present in the received telegraph signals, also show speed differences between the sending and receiving machines. The effect of a slow sending speed is to cause each unit to be greater than 22 milliseconds and each transition to occur progressively later than it should. The effect on the margin of operation is to raise both limits, the lower limit being raised much more than the upper limit. For example, a margin of 35 to 100 indicates the sending speed is five per cent slow. On the other hand, the effect of a fast sending speed is to cause each unit to be smaller than 22 milliseconds and each transition to occur progressively earlier than it should. The effect on the margin of operation is to lower both limits, the upper limit being lowered much more than the lower limit. For example, a margin of 5 to 60 indicates the sending speed is five per cent fast.

## 97. Telegraph Transmission Measuring Set

In order to make a more detailed analysis of distortion and to determine such factors as the extent of the displacement of received transitions, a time interval measuring device must be used. A telegraph transmission measuring set has been standardized for this purpose. It employs the condenser charge principle, in conjunction with a brush type distributor and associated equipment, for the measurement of maximum or total distortion, as well as bias, in terms of per cent length of a unit interval signal.

This measuring set may be compared to the teletypewriter when used as a measuring device, on the basis that each has a receiving distributor, receiving relays and associated equipment, and operates from teletypewriter signals. The essential difference is that the teletypewriter measurement depends upon shifting the range finder mechanism for the presence or non-presence of errors in the copy, and the resultant range of shifting is used as a measure of distortion. The measuring set accomplishes the same result automatically and gives direct meter readings of per cent distortion. Of course, the transmission measuring set will also serve several other purposes.

In order to consider the underlying operating principle of this measuring set, let us refer to Figure 190-A where a series circuit consisting of a resistance,  $R$ , condenser,  $C$ , battery and key is shown. At the instant the key is closed, the condenser will present an opposition to the initial current flow. There will continue to be a flow of electricity into the condenser until its voltage rises to be equal and opposite that of the battery. The speed with which the condenser voltage rises depends upon the resistance in series with the battery.

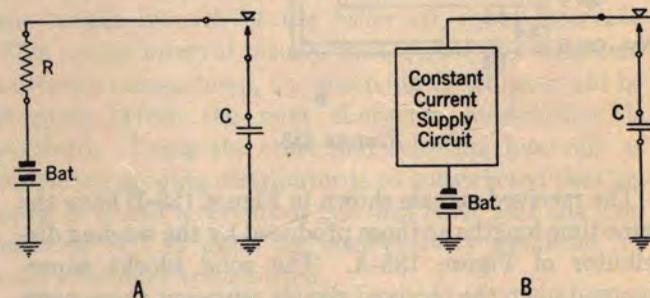


FIGURE 190

If it requires one second to rise to four-eighths of the battery voltage, it will rise to six-eighths in two seconds, to seven-eighths in three seconds, to fifteen-sixteenths in four seconds, etc. If this condenser charge were interrupted and some means used to measure the condenser voltage, it could be determined from the above just how much time had elapsed since the key was closed.

In the measuring set, the time-charge relationship is simplified by arranging a vacuum tube circuit which has a constant output current to charge the condenser, so that the voltage increase is the same for each millisecond of time that the condenser is allowed to charge. This is known as a "constant current supply circuit" and is connected in the circuit as schematically indicated in Figure 190-B.

From the simplified drawing of Figure 191 it may be noted that a polar relay is used as a master relay to repeat the incoming teletypewriter signals into a simple form of one-way polar circuit, in which are located three other polar relays. One of the three closes the start magnet circuit on every mark-to-space transition to release the brush arm of the distributor whenever the first M-S transition of a character is received. The second one of the three relays is connected to a measuring condenser,  $C_1$ , to interrupt its charge on each M-S transition and transfer the condenser to a voltage indicating circuit. The third relay performs the same function with another measuring condenser,  $C_2$ , on each S-M transition.

The relays thus serve to interrupt the condenser charging at the desired time and to immediately transfer the charged condensers to a circuit which will measure the voltage existing across their terminals. In addition, some arrangement must be used to discharge the condensers when they are transferred back to the charging circuit, so that they may start charging again at the right time. To do this, a segmented ring type of distributor is used. The ring consists of fourteen alternate long and short segments, the long segments being three times the length of the short ones.

The speed of the governed motor is adjusted so that it takes 22 milliseconds for the brush arm to pass over a long segment and the adjacent short one.

The short segments are all connected together to ground, but there are no connections to the long segments with the exception of the one on which the brush rests in its stopped position. To this Stop segment is connected what is known as a "Stop Compensator Voltage" which can be varied by means of a potentiometer, the purpose of which will be indicated later. The brush arm is connected to the ungrounded side of the Constant Current Supply so that when the brush arm is resting on a short grounded segment, the condenser connected to the Constant Current Supply Circuit will be completely discharged and will be allowed to start charging as soon as the brush arm moves off the grounded segment.

The segmented ring is oriented so that, when perfect teletypewriter signals are being received, the brush arm is just half way between two successive short grounded segments when any transition occurs. Since it requires seventy-five per cent of 22 milliseconds for the brush arm to pass over a long segment, that segment is said to be seventy-five per cent in length. The charging current is adjusted to such a value that when the measuring condenser starts charging as the brush leaves a short grounded segment, the voltage across its terminals will rise to about 55 volts at the instant that the brush reaches the position midway between two successive segments.

Whenever a perfect transition occurs, a relay interrupts the charging of the measuring condenser and transfers it to the circuit containing the bias meter and

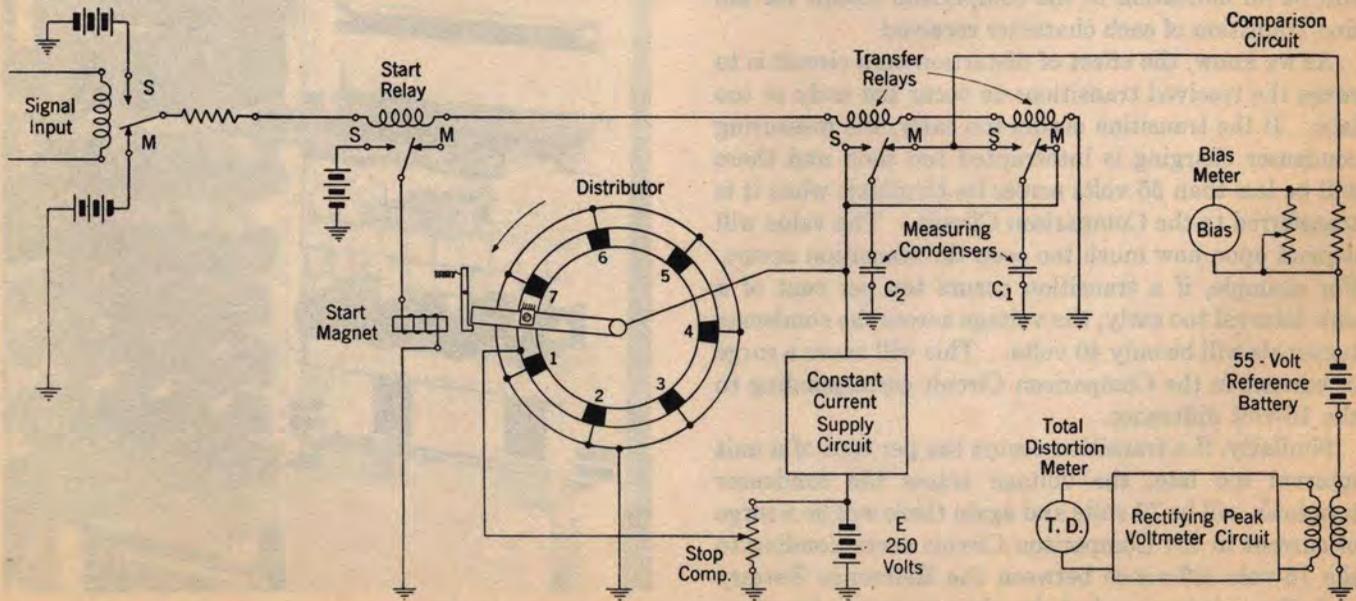


FIG. 191. TELEGRAPH TRANSMISSION MEASURING SET

reference battery just as the voltage across its terminals has reached 55 volts. The circuit containing the bias meter and 55-volt reference battery is known as the "voltage indicating circuit". If there is no transition when the brush is passing over a blank segment the condenser remains connected to the charging circuit and is charged up to 110 volts, then discharged by the short grounded segment, thereby having no effect in the voltage indicating circuit.

With the 55-volt battery in the voltage indicating circuit poled to oppose the condenser voltage, there will be no flow of current if the voltage of the measuring condenser is also 55 volts when transferred to this circuit by a relay. If the condenser voltage is greater than the battery voltage, there will be a discharge of electricity from the condenser through the opposing battery; but for a condenser voltage less than 55, the battery will cause a current flow into the condenser. The greater the voltage difference, the greater the surge of current. Since the condenser voltage is being compared to the battery, this voltage indicating circuit may be considered as a Comparison Circuit and the battery as a Reference Battery.

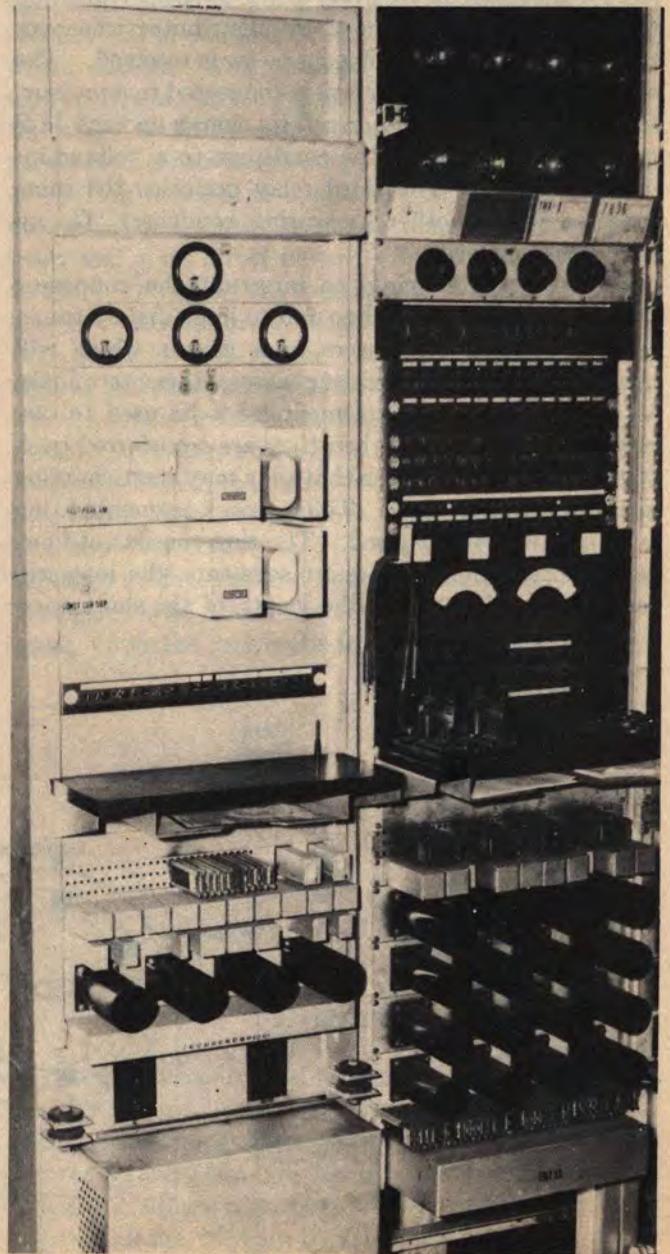
Since the reference point is the first M-S transition, and the brush does not rest on a grounded segment in its stopped position, it is necessary to control the voltage across the condenser  $C_1$  so that there will be no indication in the comparison circuit when this first M-S transition occurs. By adjusting the "stop compensator" potentiometer associated with the battery supply  $E$  so that the voltage to ground is always equal to that of the reference battery, condenser  $C_1$  will be charged to the reference battery voltage each time the brush passes over the "Stop" segment. Accordingly, there will be no indication in the comparison circuit for the first transition of each character received.

As we know, the effect of distortion on a circuit is to cause the received transitions to occur too early or too late. If the transition occurs too early, the measuring condenser charging is interrupted too soon and there will be less than 55 volts across its terminals when it is transferred to the Comparison Circuit. The value will depend upon how much too soon the transition occurs. For example, if a transition occurs ten per cent of a unit interval too early, the voltage across the condenser terminals will be only 40 volts. This will cause a surge of current in the Comparison Circuit corresponding to the 15-volt difference.

Similarly, if a transition occurs ten per cent of a unit interval too late, the voltage across the condenser terminals will be 70 volts and again there will be a surge of current in the Comparison Circuit corresponding to the 15-volt difference between the Reference Battery and the condenser, but in the opposite direction. With perfect transitions received, however, the voltage

across the terminals of the measuring condensers will always be 55 volts at the instants that a relay interrupts the charging and transfers them to the Comparison Circuit. In this case there will be no current flow in the indicating circuit.

In order to determine the amount of any distortion present in the signals, some arrangement must be used to measure the momentary voltage differences. The ordinary voltmeter would not be satisfactory because the voltmeter needle would not have enough time to reach a steady reading. A special vacuum tube circuit known as a "rectifying peak voltmeter circuit" is used for this purpose. The indicating meter included in this



118-A-1 TELEGRAPH TRANSMISSION MEASURING SET (LEFT)  
ADJACENT TO VOICE-FREQUENCY CARRIER BAY

circuit then reads the total distortion present as a percentage of unit signal.

A "bias meter" (scale 25-0-25) is connected in series with the 55-volt reference battery. With spacing bias in the received signals, all of the space-to-mark transitions occur later than they should, allowing condenser  $C_2$  to rise to a voltage higher than 55 volts; therefore, there will be a discharge current out of the condenser of the same magnitude each time a space-to-mark transition transfers  $C_2$  to the Comparison Circuit. These discharge currents will cause the meter needle to swing to the spacing side of zero. The larger the spacing bias the larger the discharge currents will be and the farther to the left the meter needle will swing. With marking bias in the received signals, all of the space-to-mark transitions will occur too early, preventing condenser  $C_2$  from rising to 55 volts. Therefore, there will be a charging current in the opposite direction from the reference battery into the condenser each time a space-to-mark transition occurs, causing the meter needle to swing to the marking side of zero an amount depending upon the magnitude of the marking bias.

A variable shunt is provided across the bias meter so that the position of the meter needle can be made to read directly the per cent bias. The meter shunt is adjusted to read correctly on four transitions per character, since the average number of transitions in miscellaneous signals is four. On two transition characters, it will read half as much as it should; on four transition characters, it will read correctly; and on six transition characters, it will read one and one-half times what it should.

For miscellaneous 60 speed teletypewriter signals where only bias exists, the average indication of the bias meter will be spacing or marking and the total distortion meter will read the same magnitude as the bias meter. However, the bias meter needle fluctuates in accordance with the number of transitions in the signals while the total distortion meter reading is steady. As noted above, the reading of the bias meter will depend upon the number of transitions per character in the signals being received.

With distortion other than bias in the signals, the bias meter needle will fluctuate over a wide range but its average position will be zero. On the other hand, the total distortion meter will give a steady reading of the maximum distortion present, but the observation must obviously be made over a period of time to obtain an accurate indication.

With both bias and other forms of distortion in the miscellaneous teletypewriter signals, the bias meter needle will fluctuate over a wide range to the right or left of zero but its average position will still give a fair indication of the bias present. The total distortion meter will indicate the sum of the bias and other forms of distortion with a steady reading of the maximum distortion present. Observation over a period of time is required to estimate the bias meter average reading, as well as to obtain an accurate indication of the maximum distortion. The readings are usually recorded with the total distortion meter reading first, followed by the sign and magnitude of the average bias meter reading,—thus 15M10, meaning 15% total distortion and 10% marking bias.

## CHAPTER XV

### ALTERNATING CURRENTS

#### 98. Source of Alternating E.M.F.

In taking up the study of alternating-current flow, we shall follow closely the same course as was followed in the study of direct currents. The theory will precede the applications, and step by step we shall pass

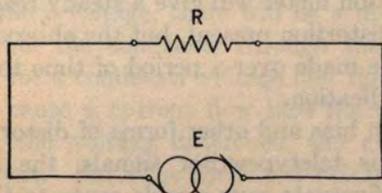


FIGURE 192

from the simple circuit to the network, from the network to the transmission of electrical energy, and thence to our ultimate aim, which is the application of these to the transmission of human speech. But along with this procedure, we shall study wherein the nature of alternating-current work differs from that of direct-current work. Perhaps the first such difference lies in the source of E.M.F.

Figure 192 represents an alternating-current cir-

cuit in its simplest form. In this figure we have a new convention for source of E.M.F., which represents the collector rings of a generator. Unlike the battery or other simple form of direct E.M.F., we cannot describe such a source of E.M.F. by simply giving its

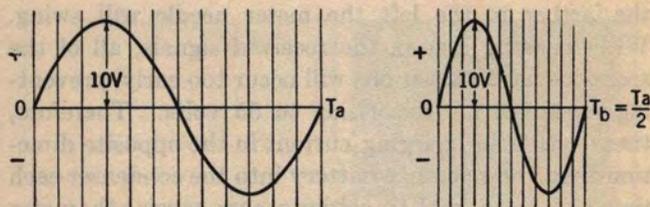


FIG. 194. SINE WAVES OF DIFFERENT FREQUENCY

voltage, for example  $E = 10$  volts. Here we have a voltage gradually increasing to a maximum value, and then decreasing to zero, to again increase to a maximum value in the opposite direction, and again decrease to zero, where the cycle repeats itself. Even if we knew the maximum voltage value, we should not know the trend of the successive values from zero to the maximum value. Figure 193 illustrates cycles of alternating E.M.F.'s all very different in this respect.

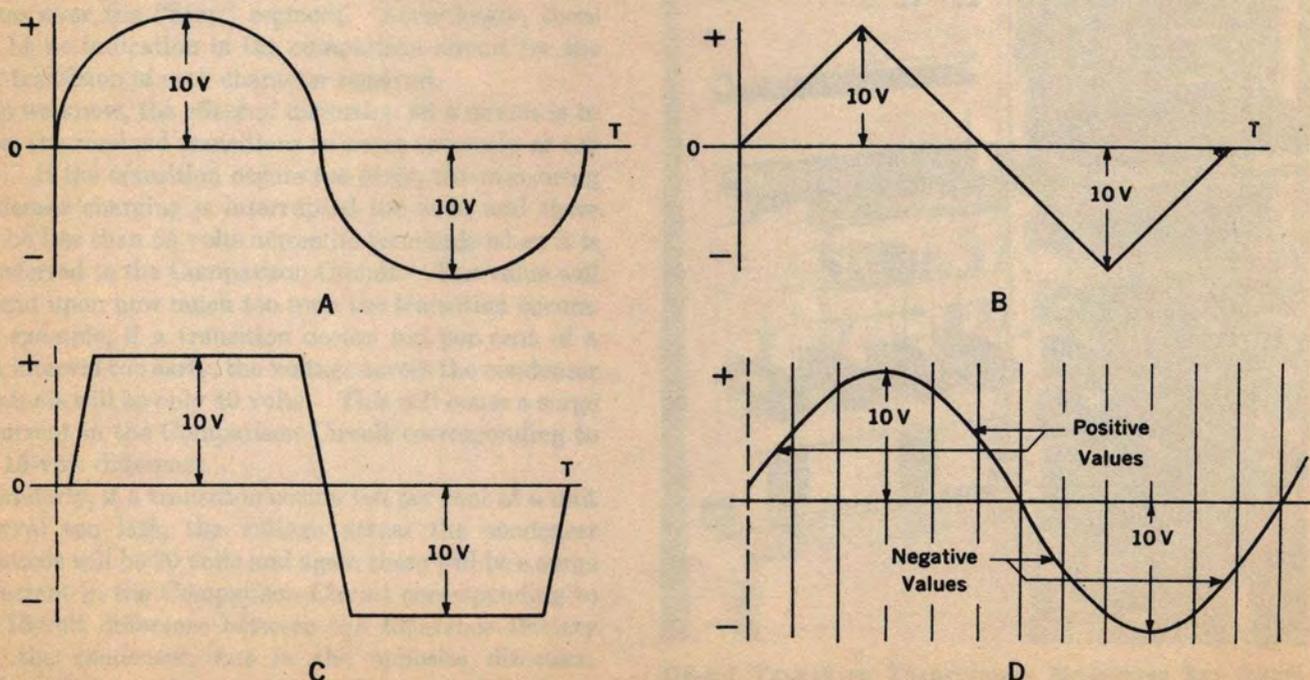


FIG. 193. THEORETICAL ALTERNATING CURRENT WAVE SHAPES COMPARED WITH SINE WAVE

Furthermore, we should not know the rapidity with which the alternations are taking place. For example, Figure 194 represents two cycles of identical E.M.F. values, but in one case the cycle is completed in one-half the time required for the other. Therefore to describe electrically a source of alternating E.M.F. we must know the following:

- a. The wave shape of the alternating cycle.
- b. The value of the E.M.F. at some specified point in the cycle.
- c. The length of time to complete the cycle, or the frequency.

In classifying electrical currents in Chapter VIII, we named two steady state conditions for alternating current; one where the wave shape is a sine wave and the other where the wave shape is not a sine wave but a complex wave. The basic study of alternating-current circuits deals only with sine waves. Complex

waves are analyzed into sine waves, just as complex tones are analyzed into **fundamentals and harmonics** (see Appendix IV).

### 99. The Sine Wave

The sine wave is named from a trigonometric function of an angle. We have learned how it may be constructed graphically, and we may treat it as a "pattern" having a name with a mathematical origin to which an E.M.F. or current may or may not conform, rather than as a mathematical expression requiring a thorough knowledge of trigonometry for interpretation. It has interesting properties and is the natural wave form in all vibratory motion. It greatly simplifies alternating-current circuits because—a **sine wave E.M.F. impressed upon a circuit having a network of any number and arrangement of resistances, induc-**

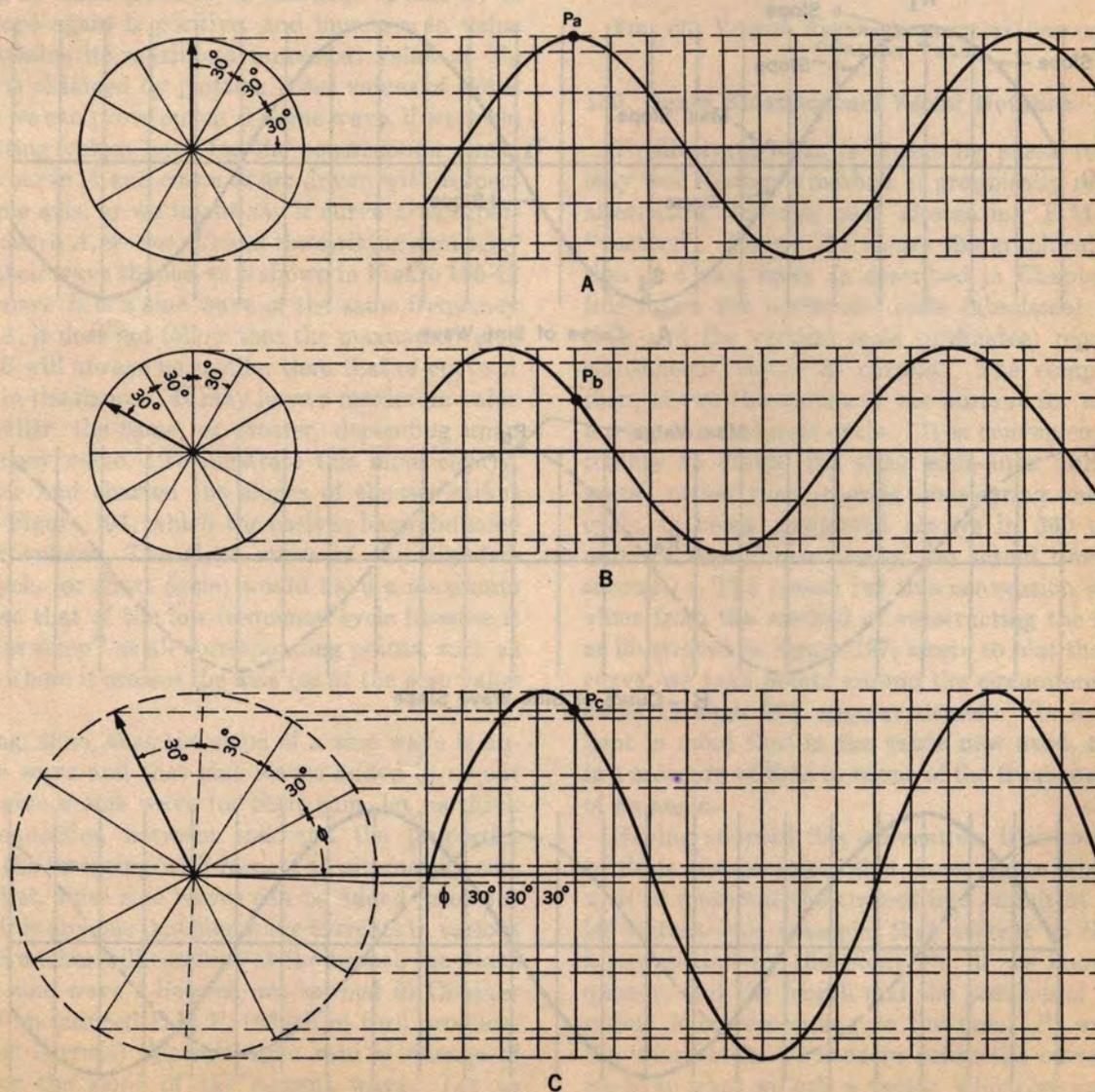


FIG. 195. GRAPHICAL PROOF THAT THE SUM OF TWO SINE WAVES IS A SINE WAVE

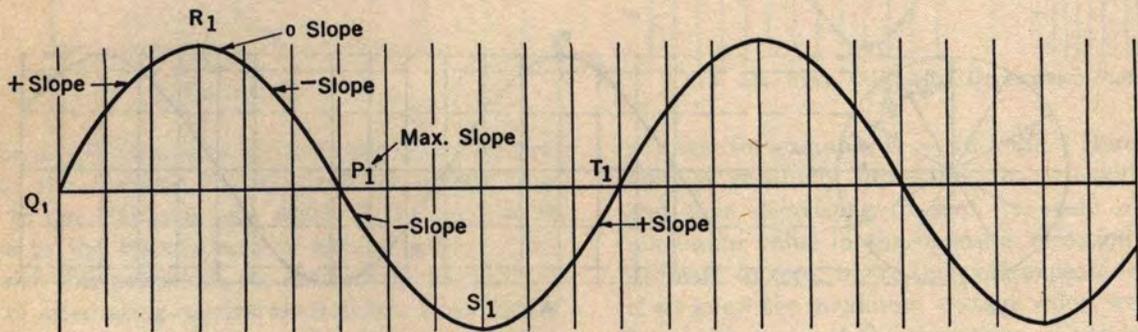
tances, and capacities with fixed values, will set up a sine wave current in every branch of the network. No other wave shape (excepting that of direct current) will give the same wave shape for the current as that for the impressed E.M.F.

The above rule holds in all its applications since the sine wave possesses the following properties:

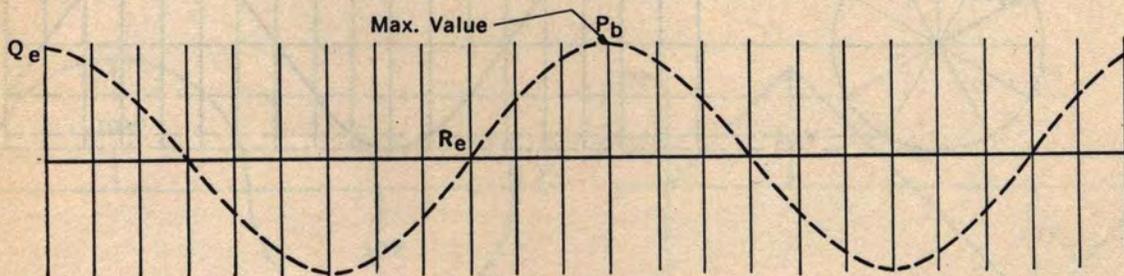
- a. Sine waves of the same frequency can be added (or subtracted) either in or out of "phase" and the wave shape of the result will be a sine wave. (Phase relations are defined in the next article.)
- b. A sine wave E.M.F. across a resistance, inductance or capacity gives a sine wave current through the resistance, inductance or capacity (though not necessarily in phase).

- c. Whenever an E.M.F. is induced on account of the ever-changing value of a sine wave current, this induced E.M.F. is a sine wave (though not in phase).

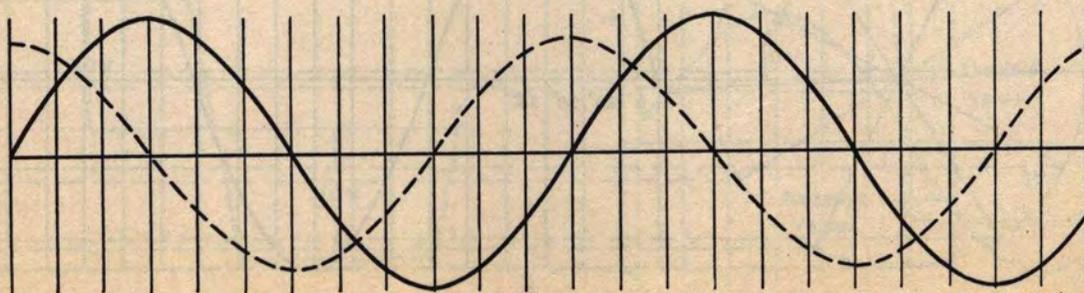
A graphical proof of property *a* may be had by referring to Figure 195. Here *A* shows a sine wave constructed graphically in the manner explained in connection with Figure 73. *B* is a similar sine wave of the same frequency but constructed independently of *A*. Now if each value on the *A* curve, for example that represented by  $P_a$ , is added to each corresponding value on the *B* curve, for example that represented by  $P_b$ , the resultant curve will be that shown as *C*. An inspection of *C* will show that it too is a sine wave and can be proven so by constructing a circle from values of the curve projected back. This is, of course, the



A - Curve of Sine Wave



B - Curve of Sine Wave Slope



C - Curves A & B Compared

FIG. 196. GRAPHICAL PROOF THAT THE SLOPE OF A SINE WAVE IS A SINE WAVE

converse of the construction of the sine wave and proof of the wave shape, since there can be only one circumference drawn through the projected points of intersection with the radii of the circle.

The properties of a sine wave given under  $b$  and  $c$  in the foregoing can be demonstrated graphically by determining the rate of change or "slope" of a sine wave at various points and plotting the successive values of the slope as shown in Figure 196-B. When the value shown by the curve in Figure 196-A is zero, as at point  $Q_1$ , the rate of change or slope is greatest. At any point between  $Q_1$  and  $R_1$ , the slope is positive and is decreasing to its minimum value, zero, at point  $R_1$ . Between  $R_1$  and  $P_1$  the slope is negative and is increasing, attaining its maximum numerical value at  $P_1$ . After passing through  $P_1$ , the slope again decreases in magnitude, but is still negative, remaining so until point  $S_1$  is reached. From  $S_1$  to  $T_1$  the slope again is positive, and increases in value until it attains its maximum numerical value at  $T_1$ . Curve  $B$  is obtained by plotting these values of slope. As before we can prove curve  $B$  a sine wave, if we wish, by projecting values back for the construction circle, but if the curve  $A$  and curve  $B$  are drawn with respect to the same axis, or we might say if curve  $B$  is superposed on curve  $A$ , we see offhand the striking similarity between their wave shapes, as is shown in Figure 196-C. Though curve  $B$  is a sine wave of the same frequency as curve  $A$ , it does not follow that the maximum value of curve  $B$  will always be smaller than that of curve  $A$  as shown in the figure. It may have a maximum value either smaller, the same, or greater, depending upon the frequency value. To illustrate this more clearly, suppose we had charted the slopes of the two curves shown in Figure 194, which themselves have the same maximum values. The slope curve of the high-frequency cycle (or short cycle) would have a maximum value twice that of the low-frequency cycle because it is "twice as steep" at all corresponding points, such as the point where it crosses the axis (or at the zero value point).

Granting, then, that the slope of a sine wave is another sine wave and that sine waves added in or out of phase give a sine wave for their sum, let us think of the connection between this and the properties stated in the foregoing with respect to alternating currents. First, since sine waves can be added in or out of phase, it is obvious that sine wave currents in various network branches will combine at the branch junctions to give a sine wave. Second, we learned in Chapter VIII that an induced E.M.F. (which in turn produces an induced current) depends upon rate of change of current, or the slope of the current wave. Let us assume a network of inductances with a sine wave

E.M.F. impressed. In each individual inductance there is a sine wave induced E.M.F., if the current is a sine wave. But there will be a sine wave current because the induced current adds to or subtracts from the current due to the impressed E.M.F., and the sum or difference of two sine waves is a sine wave. Thus we can analyze all the currents in the branches of any network by either an application of sine wave addition and subtraction, sine wave slopes, or a combination of the two properties.

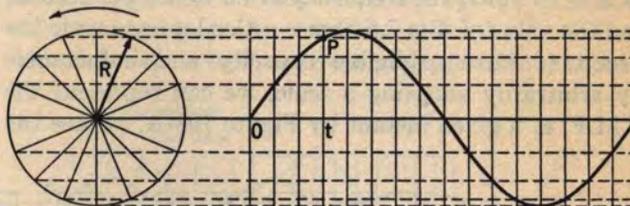


FIG. 197. VECTOR REPRESENTATION OF INSTANTANEOUS CURRENT VALUE

## 100. Phase Relations and Vector Notation

To illustrate what is meant by phase relation, we may well discuss a method of graphically representing alternating currents and alternating E.M.F.'s with "vectors". Figure 197 shows the graphical construction of a sine wave as described in Chapter VI. In this figure the horizontal scale (abscissae) represents time and the vertical scale (ordinates) represents instantaneous values of current. The complete curve then, shows the values of the current for all instants during one complete cycle. It is convenient and customary to divide the time scale into units of "degrees" rather than seconds, considering one complete cycle as being completed always in 360 degrees or units of time (regardless of the actual time taken in seconds). The reason for this convention will be obvious from the method of constructing the sine curve as illustrated in Figure 197, where to plot the complete curve, we take points around the circumference of the circle through 360 angular degrees. It needs to be kept in mind that **in the sense now used, the degree is a measure of time in terms of the frequency, and not of an angle.**

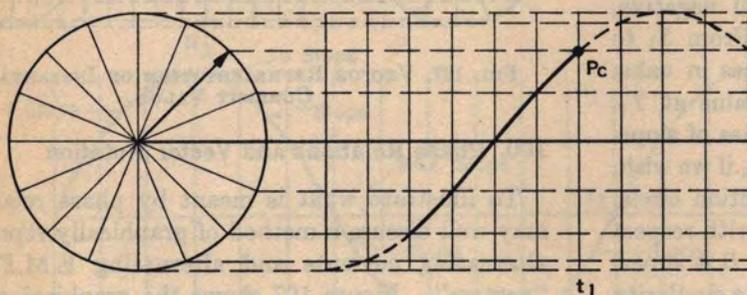
Having adopted this convention, it is not necessary to draw the complete sine curve figure whenever we wish to represent the current in a circuit at a particular instant—for example, that current at the instant  $t$ , represented by the point  $P$ . If we know the frequency, and the length and the position of the single radius  $R$  corresponding to the point  $P$ , we have all the information we need to define the current. Here we have what we call a vector, which we can imagine as a radius of the circle, having a length equal to the

maximum current or E.M.F. value of the sine wave in question. The angle this vector makes with the horizontal gives the position of point *P* and if we assume a direction of rotation for the vector, we can always determine by the position of the vector whether the value of the current or E.M.F. is increasing or decreasing, and its direction. The accepted convention for direction of rotation is counterclockwise and will be understood hereafter, without the arrow being used to indicate it.

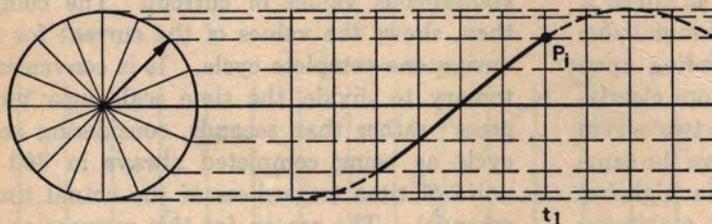
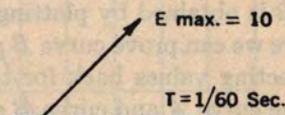
In Figure 192, let us assume that the maximum value of *E* is 10 volts, the frequency is 60 cycles per second, and the value of *R* is 7.5 ohms. Also let us assume the circuit to have negligible capacity and inductance. By arbitrarily adopting a scale, we can represent the E.M.F. at a given instant by Figure 198-A. Since the

inductance and capacity of the circuit are negligible, the current at the corresponding instant will neither be retarded by inductance nor have a component part required to "charge" the circuit. It will be that determined solely by Ohm's Law. Consequently, it will change in value as the E.M.F. changes in value. In other words, it will "keep in step", becoming a maximum of 1.33 amperes at exactly the same time that the E.M.F. becomes a maximum of 10 volts, and becoming zero at exactly the same time that the E.M.F. becomes zero. The conventional expression to describe this time relation between the voltage and the current is that the voltage and current are in "phase".

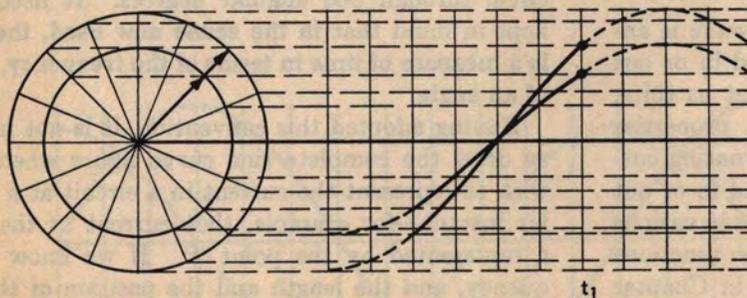
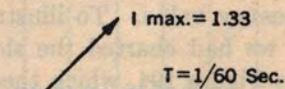
But if, instead of a circuit such as that shown by Figure 192, we have the circuit shown by Figure 199, it will be necessary to consider the effect of the in



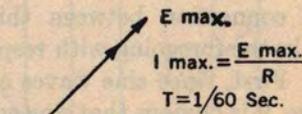
A - Voltage



B - Current



C - Current and Voltage



ACTUAL PICTURES

VECTOR REPRESENTATION

FIG. 198. CURRENT AND VOLTAGE IN PHASE

ductance. This reacts to any change in current value, and an alternating current is changing in value at all times. We should therefore expect the inductance to materially affect the value of the current and to throw the maximum points out of step, or phase, because the maximum value of current will not have been established until some time after the E.M.F. has reached

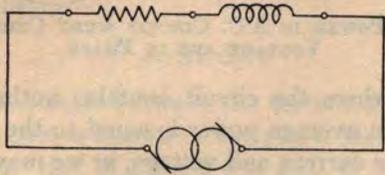


FIGURE 199

its maximum value. Figure 200 represents the relation of voltage and current that are out of phase due to the circuit having inductance. Here the vectorial representation must show the extent to which the voltage and current are out of phase. This is accomplished by having the voltage vector ahead of the current vector in its rotation by an angle which is a measure of the time by which the current "lags" behind the voltage, and whose value is obvious from the relative position of the radii of the two circles.

In the case of a circuit having a series condenser instead of an inductance, the circuit reactions are the reverse. The current vector then is ahead of, or "leads", the E.M.F. vector as shown by Figure 201. Electrical conditions in circuits containing inductance or capacity, therefore, can be represented by current and voltage vectors, which will, in general, be out of phase. Moreover, in dealing with complex networks containing inductance or capacity, we encounter current vectors which are out of phase not only with their voltage vectors, but with each other.

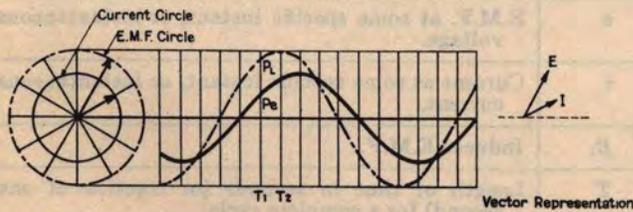


FIG. 200. CURRENT LAGGING IMPRESSED E.M.F.

In direct-current networks, we used equations based on Kirchoff's Laws which called for adding or subtracting current or E.M.F. values. In alternating-current work, we cannot accomplish this by merely adding the numerical lengths of the vectors. We must instead combine them in such a manner as to take into consideration any phase differences that may exist. This may be done graphically by placing the vectors to be added end to end, and drawing a line from the

butt of the first arrow to the tip of the last. This line, called the resultant, is a vector which gives the magnitude and phase of the sum. For example, let us assume that it is desired to find the current delivered by the generator of Figure 202, when the currents in the parallel branches have the values and phase relationships indicated by vectors 1, 2, and 3. These vectors are placed end to end and the resultant drawn as indicated in 4. The length of this resultant vector gives the value of the current delivered by the generator and its angular position indicates its phase relationship with respect to the current in the parallel branches.

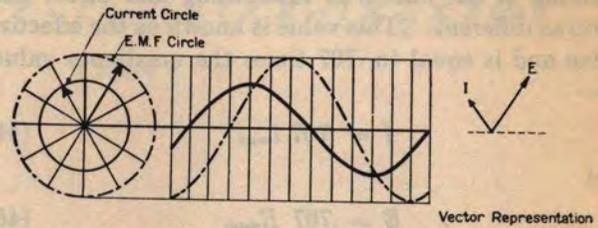


FIG. 201. CURRENT LEADING IMPRESSED E.M.F.

### 101. Effective E.M.F. and Current Values

In laying out current and voltage vectors thus far, we have indicated in each case the current or voltage at some particular instant of time in its cycle. The length of the vector gave the maximum value of the current or voltage and the angle that the vector made with the horizontal, in a counterclockwise sense, indicated the particular instant being considered.

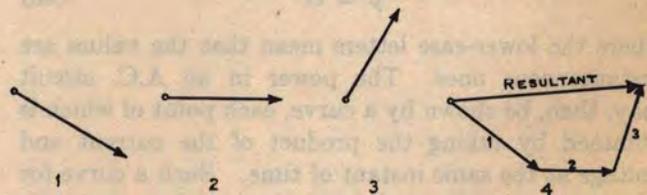
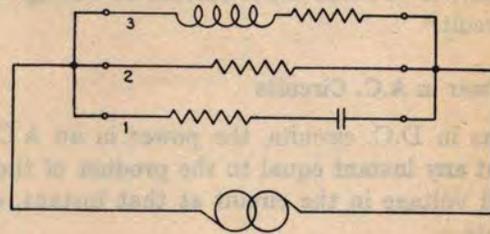


FIG. 202. GRAPHICAL ADDITION OF CURRENT VECTORS

For practical purposes, however, it would be inconvenient to be always under the necessity of stating both a value and a position in time in defining an alternating current or voltage. It is advantageous, rather, to adopt some arbitrary standard so that only the value of the current or voltage need be given to

define it, its position in time being understood from the convention adopted. The maximum value would perhaps appear to be the logical choice, but this has certain disadvantages. Another, and more useful value would be the average value over a complete half-cycle, this being equal for the sine wave to .636 times the maximum value.

Still more useful is a value so selected that the heating effect of a given value of alternating current in a resistance will be exactly the same as the heating effect of the same value of direct current in the same resistance. The advantage of such a convention is apparent, since it obviates to a degree the necessity for thinking of the effects of alternating and direct currents as different. This value is known as the **effective value** and is equal to .707 times the maximum value, or—

$$I = .707 I_{max}. \quad (44)$$

and

$$E = .707 E_{max}. \quad (45)$$

where  $E$  and  $I$  without subscripts indicate effective values. Unless specifically stated otherwise, values of alternating currents and voltages are always given in terms of their effective values. Likewise, vectors representing currents and voltages give the effective value of the current or voltage by their length and, unlike the vectors we have previously considered, do not indicate by their angular position a particular instant of time within the cycle but only the time relationship of the current and voltage with reference to each other, or to some other current or voltage in the same circuit.

## 102. Power in A.C. Circuits

Just as in D.C. circuits, the power in an A.C. circuit is at any instant equal to the product of the current and voltage in the circuit at that instant, or we may write—

$$p = ei \quad (46)$$

where the lower-case letters mean that the values are instantaneous ones. The power in an A.C. circuit may, then, be shown by a curve, each point of which is obtained by taking the product of the current and voltage at the same instant of time. Such a curve for the case where the current and voltage in a circuit are in phase is shown by Figure 203.

It will be noted that, since the current and voltage are both negative at the same time, the power loops are both positive, which means that no power is being returned from the circuit to the generator. In other words, all of the power delivered by the generator is being absorbed in the resistance of the circuit. For

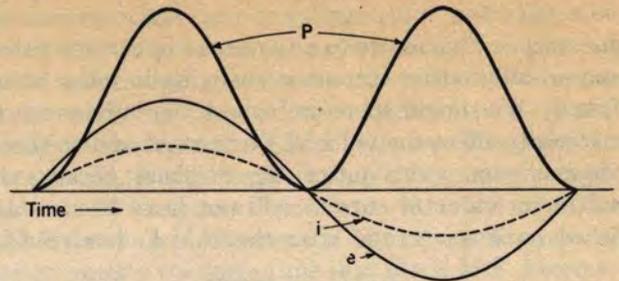


FIG. 203. POWER IN A.C. CIRCUIT WHEN CURRENT AND VOLTAGE ARE IN PHASE

this case, where the circuit contains nothing but resistance, the average power is equal to the product of the effective current and voltage, or we may write—

$$P = EI \quad (47)$$

and, as always,—

$$P = I^2 R \quad (48)$$

The condition where the circuit contains either inductance or capacity in addition to resistance, and the current and voltage are accordingly not in phase, is somewhat different. The power curve for such a case is shown by Figure 204. Here the product  $ei$  gives both positive and negative values and we have the

TABLE VII  
CONVENTIONAL SYMBOLS USED IN ALTERNATING-CURRENT WORK

SYMBOL	STANDS FOR
$P$	Average power for a cycle of E.M.F. and current.
$E$	Effective E.M.F.
$I$	Effective current.
$E_{ave}$	Average E.M.F.
$I_{ave}$	Average current.
$e$	E.M.F. at some specific instant, or instantaneous voltage.
$i$	Current at some specific instant, or instantaneous current.
$E_1$	Induced E.M.F.
$T$	Length of time in seconds (or fraction of one second) for a complete cycle.
$f$	Frequency or the number of cycles per second.
$Z$	Impedance in ohms.
$X_L$	Inductive reactance in ohms.
$X_C$	Capacity reactance in ohms.
$X$	Total reactance in ohms.
$Y$	Admittance in mhos.
$\theta$	Angle between current and impressed E.M.F., or between impedance and resistance, etc.

positive power loops *A* and *B* and the negative loops *C* and *D*. The latter loops represent power returned to the generator from the circuit. The total power absorbed by the circuit is obviously equal to the sum of *A* and *B* minus the sum of *C* and *D*. In this case, then, the power, *P* is no longer equal to *EI* but to something less than that. The factor by which *EI* must be reduced to obtain the true power is determined by the phase relation between the current and voltage, this power being—

$$P = EI \cos \theta \quad (49)$$

where  $\theta$  is the angle between the current and voltage. The term,  $\cos \theta$ , is known as the **power factor** and has a maximum value of 1 when  $\theta$  is zero, or the current and voltage are in phase.

It may be noted that the expression,  $P = I^2R$ , re-

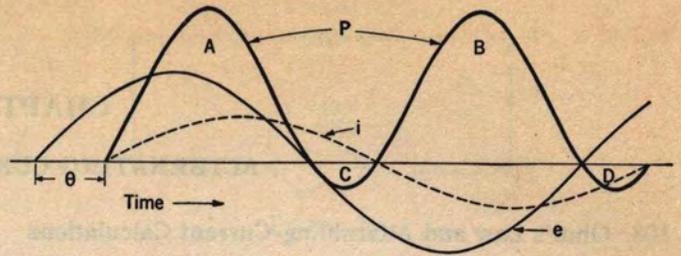


FIG. 204. POWER IN A.C. CIRCUIT WHEN CURRENT AND VOLTAGE ARE NOT IN PHASE

mains true in this case and conforms with Equation (49) because as we shall learn in the next chapter,  $R = Z \cos \theta$  and  $I = E/Z$ , from whence—

$$\begin{aligned} P &= I^2R = I \times I \times R = I \times \frac{E}{Z} \times Z \cos \theta \\ &= IE \cos \theta. \end{aligned}$$

## CHAPTER XVI

### ALTERNATING CURRENTS—(Continued)

#### 103. Ohm's Law and Alternating-Current Calculations

In Chapter I we learned that the relation between the voltage and the current in a D.C. circuit was expressed by Ohm's Law, or

$$\frac{E \text{ (volts)}}{I \text{ (amperes)}} = R \text{ (ohms)}$$

We found this expression indispensable in our study of direct-current circuits, and certainly we shall want to apply it to alternating-current circuit calculations if we can. On the other hand, we have learned of circuit properties other than resistance that influence alternating-current flow. Moreover, these properties, viz., capacity and inductance, not only change the value of the current in amperes but introduce changes in the phase relation of the current to the voltage. Again, the effects of inductance and capacity depend entirely upon the particular frequency which we wish to consider. We must therefore introduce some new quantity that will express in ohms not only the resistance to current but the **combined effects** of resistance, capacity and inductance at a definite stated frequency. This quantity is called **impedance**, and Ohm's Law is adjusted to read—

$$Z \text{ (ohms)} = \frac{E \text{ (volts)}}{I \text{ (amperes)}} \quad (50)$$

where  $Z$  is the symbol for **impedance** or the **combined effect of the circuit's resistance, inductance and capacity taken as a single property which can be expressed in ohms for any given sine wave frequency**. It follows, then, that if we can by certain calculations reduce a circuit's resistance expressed in ohms, its inductance expressed in henrys, and its capacity expressed in microfarads, to a single expression in ohms, we can calculate the current at a given frequency in any single branch as readily as though it were a branch of a direct-current network.

The effect of inductance or capacity in opposing the flow of current in any alternating-current circuit is known as **reactance** and is expressed in ohms the same as resistance. However, in combining resistance and reactance into a single property measured in ohms, which we have already referred to as impedance, we must add them vectorially because they do not act in phase. We shall take up the calculation of impedance after first learning how the reactance may be deter-

mined for any single frequency from the inductance and capacity values in a given circuit branch.

#### 104. Inductive Reactance

Referring to Chapter VIII, it will be recalled that we considered two factors as being involved in the calculation of the effects of inductance; first, the physical property of the circuit called inductance and second, the rate of change of current value, which uses inductance "as a tool" in creating the reactive effects. In an **alternating-current circuit containing inductance, therefore, we should expect greater reactance for higher frequencies** because higher frequencies mean an increase in the average rate of change of current. By referring to Figure 205 this becomes apparent. Here are two current cycles of the same effective value but

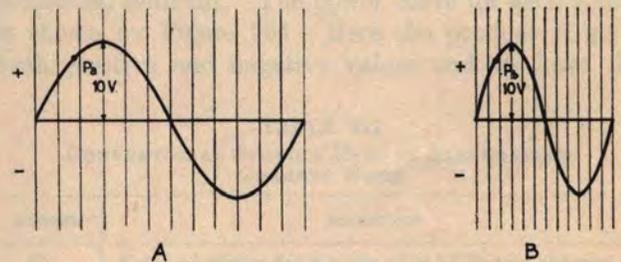


FIGURE 205

the  $A$  cycle has twice the period, or half the frequency of the  $B$  cycle. Also the slope of the  $A$  curve at any point such as  $P_a$ , is half the slope at any corresponding point such as  $P_b$  on the  $B$  curve. The slope is, as has been seen, the measure of current change and we would expect, therefore, that the induced E.M.F. of the  $B$  curve would be twice as great as that of the  $A$  curve. Thus, the reactance due to inductance depends upon first, the inductance of the circuit and second, the frequency of the current. As a matter of fact, it can be proven that the inductive reactance expressed in ohms is equal to the **inductance in henrys times the frequency in cycles per second, multiplied by  $2\pi$**  or—

$$X_L = 2\pi fL \quad (51)$$

where  $X_L$  is the inductive reactance in ohms,  $\pi$  is 3.1416,  $f$  is the frequency expressed in cycles per second, and  $L$  is the inductance in henrys.

For practical use this becomes—

$$X_L = 6.2832 fL \quad (52)$$

**Example:** In Figure 206 assume that the source of alternating E.M.F. is a sine wave, 10 volts, 1000 cycles per second, and the inductance shown has negligible resistance. What is the effective current through the inductance?

**Note:**—In practice inductance coils have appreciable resistance because any coil winding must contain a definite length of wire; the condition assumed here is that the effect of the inductance is so much greater than that of the resistance that we may neglect the value of the resistance in the calculations.

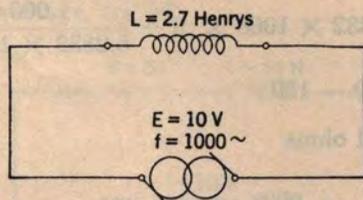


FIGURE 206

**Solution:**

$$X_L = 6.2832fL = 6.2832 \times 1000 \times 2.7$$

$$= 16964 \text{ ohms}$$

$$I = \frac{E}{16964}$$

$$= \frac{10}{16964}$$

$$= .00059 \text{ ampere, ans.}$$

In this example, the current will be  $90^\circ$  behind the impressed voltage, as shown in Figure 207, because the induced E.M.F. due to the current must be equal and opposite to the impressed E.M.F. and the induced E.M.F., as previously explained, is the rate of change or slope of current times the inductance and must, therefore, be  $90^\circ$  behind the current.

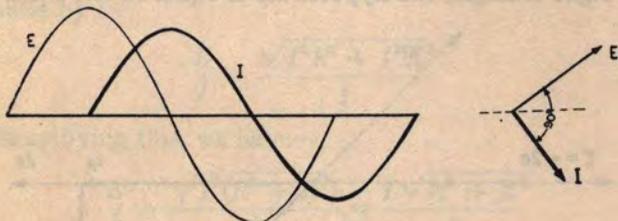


FIG. 207. EFFECT OF INDUCTIVE REACTANCE

### 105. Capacity Reactance

Capacity reactance has opposite effects to inductive reactance—in fact, the two tend to neutralize each other. Capacity reactance decreases with increasing frequency and capacity values. It also tends to make

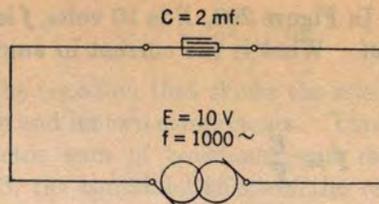


FIGURE 208

the current lead instead of lag the voltage (see Figures 207 and 209). Accordingly, if inductive reactance is assumed as positive, capacity reactance must be taken as negative.

This time relation of the voltage and current in a circuit containing capacity may be seen by referring to Figures 208 and 209. Here when the impressed voltage  $E$  is at its maximum positive value, the condenser is charged to a value equal and opposite to the impressed voltage. The current in the circuit is therefore zero. As the positive impressed voltage decreases toward zero the opposite voltage of the condenser forces current to flow in a negative direction. This negative current reaches its maximum value when the impressed voltage becomes zero. Now the impressed voltage reverses, becoming negative, and as it rises to its maximum negative value, charges the condenser in the opposite (positive) direction. During this time, the negative current decreases to zero as the condenser becomes fully charged. Then as the negative impressed voltage decreases from its maximum, the condenser voltage again takes control and causes the current to build up in the opposite direction. The relationships are therefore as shown in the figure with the current leading the voltage by  $90^\circ$ .

The equation for capacity reactance is as follows:

$$X_c = -\frac{1}{2\pi fC} \quad (53)$$

where  $C$  is capacity in farads. Converting  $C$  to the customary capacity unit, microfarad, we have—

$$X_c = -\frac{1,000,000}{2\pi fC} \quad (54)$$

or with 3.1416 substituted for  $\pi$ —

$$X_c = -\frac{1,000,000}{6.2832fC} \quad (55)$$

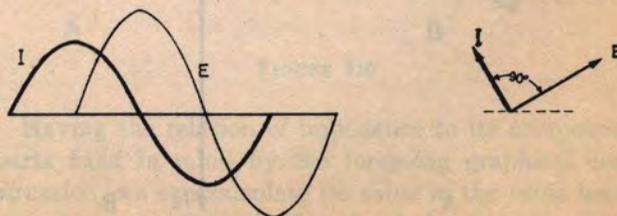


FIG. 209. EFFECT OF CAPACITY REACTANCE

**Example:** In Figure 208,  $E$  is 10 volts,  $f$  is 1000 and  $C$  is 2 mf. What is the current in amperes?

**Solution:**

$$I = \frac{E}{X_c}$$

$$X_c = -\frac{1,000,000}{6.2832 \times 1000 \times 2}$$

$$= -\frac{1,000}{6.2832 \times 2}$$

$$= -79.5 \text{ ohms}$$

$$I = -\frac{10}{79.5}$$

$$= -.126 \text{ ampere, ans.}$$

(minus sign here means leading current)

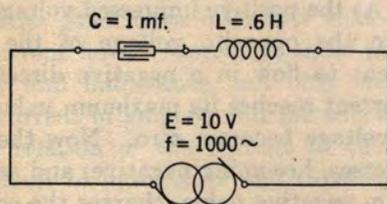


FIGURE 210

### 106. Combination of Inductive and Capacity Reactances

If we wish to get the combined or total reactance of an inductance in series with a capacity, such as that shown in Figure 210, we may combine the reactances as follows:

$$X = X_L + X_c$$

or, from formulas (52) and (55)—

$$X = 6.2832fL - \frac{1,000,000}{6.2832fC} \quad (56)$$

Here the signs take care of the neutralizing effect and if the calculated value of  $X$  is positive, the inductive reactance predominates; if negative, the capacity reactance predominates.

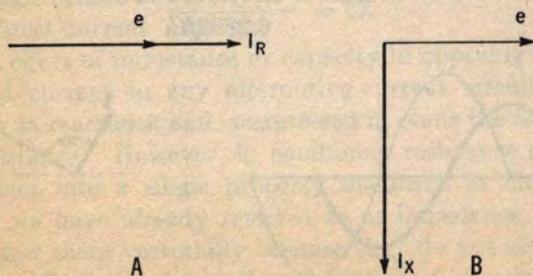


FIGURE 211

**Example:** Calculate the current in the circuit shown by Figure 210.

**Solution:**

With no resistance in the circuit—

$$I = \frac{E}{X}$$

and

$$X = X_L + X_c$$

$$= 6.2832fL - \frac{1,000,000}{6.2832fC}$$

$$X = 6.2832 \times 1000 \times .6 - \frac{1,000,000}{6.2832 \times 1000 \times 1}$$

$$= 3770 - 159$$

$$= 3611 \text{ ohms}$$

$$I = \frac{10}{3611} = .0028 \text{ ampere, ans.}$$

### 107. Impedance

To determine a way to combine reactance and resistance when we wish to evaluate the impedance, let us consider the relation between voltage and current under two conditions; first, when a circuit contains pure resistance, and second, when it contains pure reactance. Under the first condition, we can represent the current and voltage as shown in Figure 211-A, and for the second condition as shown in Figure 211-B. For the purpose of this discussion, the circuits are assumed to be such that  $I_R = I_X$ . If now we connect  $R$  and  $L$  in series and allow a voltage of  $2e$  to act on the combination, we may consider the resulting current as made up of two parts, one due to a voltage  $e$  acting on  $R$ , and the other due to a voltage  $e$  acting on  $L$ . The total current will be the sum of these two components, but the addition must be made vectorially as illustrated by Figure 212. Here since we have a right triangle, the hypotenuse is equal to the square

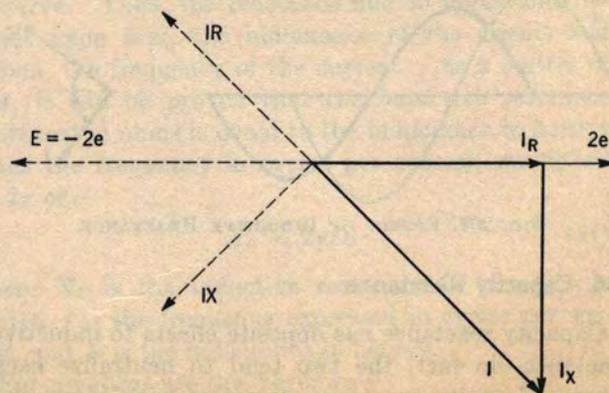


FIGURE 212

root of the sum of the squares of the two legs, or calling the components  $I_R$  and  $I_x$ , we have—

$$I = \sqrt{I_R^2 + I_x^2}$$

The voltage drop across  $R$  due to the flow of the current,  $I$ , is  $IR$ , and this drop is exactly opposite in phase with  $I$ . The drop across  $L$  is  $IX$ , with a phase relationship such that  $I$  leads  $IX$  by  $90^\circ$ . The latter will be clear if we refer again to the circuit of pure inductance pictured in Figure 206. Here the current lags the impressed voltage by  $90^\circ$ , and consequently leads the voltage drop, which is equal and opposite to the impressed voltage, by  $90^\circ$ .

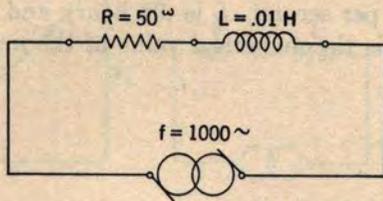


FIGURE 213

In our circuit containing both  $R$  and  $L$ , therefore, we have two component voltage drops,  $IR$ , and  $IX$ ,  $90^\circ$  out of phase, the sum of which must be equal to the total impressed E.M.F. and exactly opposite in phase. Adding these components vectorially—

$$E = \sqrt{(IR)^2 + (IX)^2} = \text{total voltage drop.}$$

$E$ , in this case, is also equal in value to the total E.M.F.  $2e$  acting upon the combined circuit.

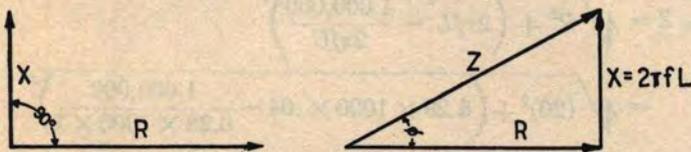


FIGURE 214

Now let us operate on this equation by dividing both sides by  $I$ —

$$\frac{E}{I} = \frac{\sqrt{I^2 R^2 + I^2 X^2}}{I}$$

Simplifying this, we have—

$$\frac{E}{I} = \frac{\sqrt{I^2(R^2 + X^2)}}{I} = \frac{I\sqrt{R^2 + X^2}}{I}$$

or—

$$\frac{E}{I} = \sqrt{R^2 + X^2}$$

However,  $\frac{E}{I} = Z$  in ohms, from Equation (50); therefore, we have—

$$Z = \sqrt{R^2 + X^2} \quad (57)$$

which is the equation that shows the relation between impedance and its two components. Thus, impedance is the vector sum of resistance and reactance. In Figure 213, the combined effect of the resistance and the reactance due to the inductance, may be represented by the vector diagram of Figure 214 in which

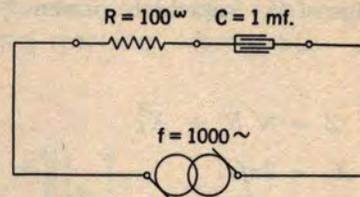


FIGURE 215

the reactance is shown as  $90^\circ$  ahead of the resistance. Similarly in Figure 215, the combined effect of resistance and capacity may be represented by the vector diagram of Figure 216 in which the reactance is shown as  $90^\circ$  behind the resistance.

In these diagrams if  $R$  is represented by the same line as the current,  $Z$  will be represented by the same line as the impressed E.M.F.; consequently the angle  $\theta$  will represent the phase difference between the voltage and current, and with the adopted convention for direction of rotation, and that for plotting time on the sinusoidal chart, will represent current lagging behind impressed E.M.F. for positive angle as shown in Figure 214, and current leading impressed E.M.F. for negative angle as shown in Figure 216.

We can now consider a simple series circuit with all three properties, or with resistance, inductance, and capacity, as shown in Figure 217. Here we have two reactances acting in opposite phase as shown in Figure 218-A. In constructing the impedance triangle,  $X_c$  must be considered as negative and subtracting from  $X_L$  as shown in Figure 218-B. If  $X_c$  is less than  $X_L$ ,  $X$  will be positive, and if  $X_c$  is greater than  $X_L$ , as shown in Figure 219,  $X$  will be negative.

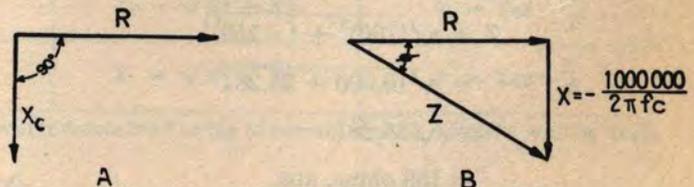


FIGURE 216

Having the relation of impedance to its component parts fixed in mind by the foregoing graphical construction, we can calculate its value in the same manner as we calculate the length of the hypotenuse of any right triangle, as has been explained. That is to

say, we square both legs and take the square root of their sum. The equation then for impedance with resistance and inductance in series (as shown by Figures 213 and 214) is:

$$Z = \sqrt{R^2 + X_L^2} \quad (58)$$

**Example:** In Figure 213,  $R = 50$  ohms,  $f = 1000$  cycles per second, and  $L = .01$  henry. What is the value of the impedance in ohms?

**Solution:**

$$\begin{aligned} Z &= \sqrt{R^2 + X_L^2} \\ X_L &= 2\pi fL \\ &= 6.2832 \times 1000 \times .01 \\ &= 62.8 \\ Z &= \sqrt{(50)^2 + (62.8)^2} \\ &= \sqrt{2500 + 3944} \\ &= \sqrt{6444} \\ &= 80.3 \text{ ohms, ans.} \end{aligned}$$

Similarly, for the impedance shown by Figures 215 and 216,

$$Z = \sqrt{R^2 + X_c^2} \quad (59)$$

**Example:** In Figure 215,  $R$  is 100 ohms,  $C$  is 1 mf. and  $f$  is 1000 cycles per second. What is the value of the impedance in ohms?

**Solution:**

$$\begin{aligned} Z &= \sqrt{R^2 + X_c^2} \\ X_c &= -\frac{1,000,000}{2\pi fC} \\ &= -\frac{1,000,000}{6.2832 \times 1000 \times 1} \\ &= -159 \text{ ohms} \\ Z &= \sqrt{(100)^2 + (-159)^2} \\ &= \sqrt{10,000 + 25,281} \\ &= \sqrt{35,281} \\ &= 188 \text{ ohms, ans.} \end{aligned}$$

The foregoing are special cases, but we may combine inductive reactance and capacity reactance in one general equation for impedance as shown by Equation (57) where

$$X = X_L + X_c = 2\pi fL - \frac{1,000,000}{2\pi fC}$$

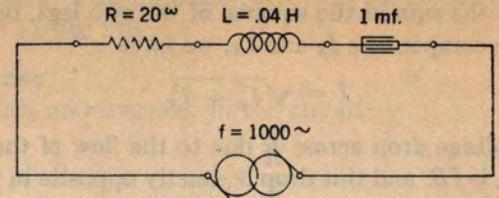


FIGURE 217

Therefore,

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1,000,000}{2\pi fC}\right)^2} \quad (60)$$

**Example:** In Figure 217,  $R$  is 20 ohms,  $f$  is 1000 cycles per second,  $L$  is .04 henry and  $C$  is 1 mf. What is the numerical value of the impedance in ohms?

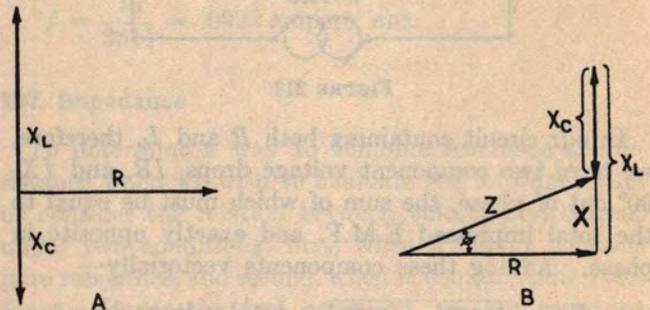


FIGURE 218

**Solution:**

$$\begin{aligned} Z &= \sqrt{R^2 + \left(2\pi fL - \frac{1,000,000}{2\pi fC}\right)^2} \\ &= \sqrt{(20)^2 + \left(6.28 \times 1000 \times .04 - \frac{1,000,000}{6.28 \times 1000 \times 1}\right)^2} \\ &= \sqrt{(20)^2 + (251 - 159)^2} \\ &= \sqrt{400 + 8464} \\ &= \sqrt{8864} \\ &= 94 \text{ ohms, ans.} \end{aligned}$$

In these calculations we have only determined the numerical value of the impedance. This does not com-

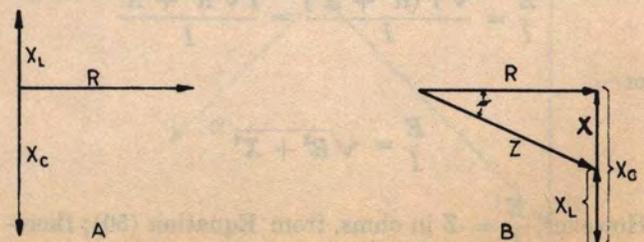
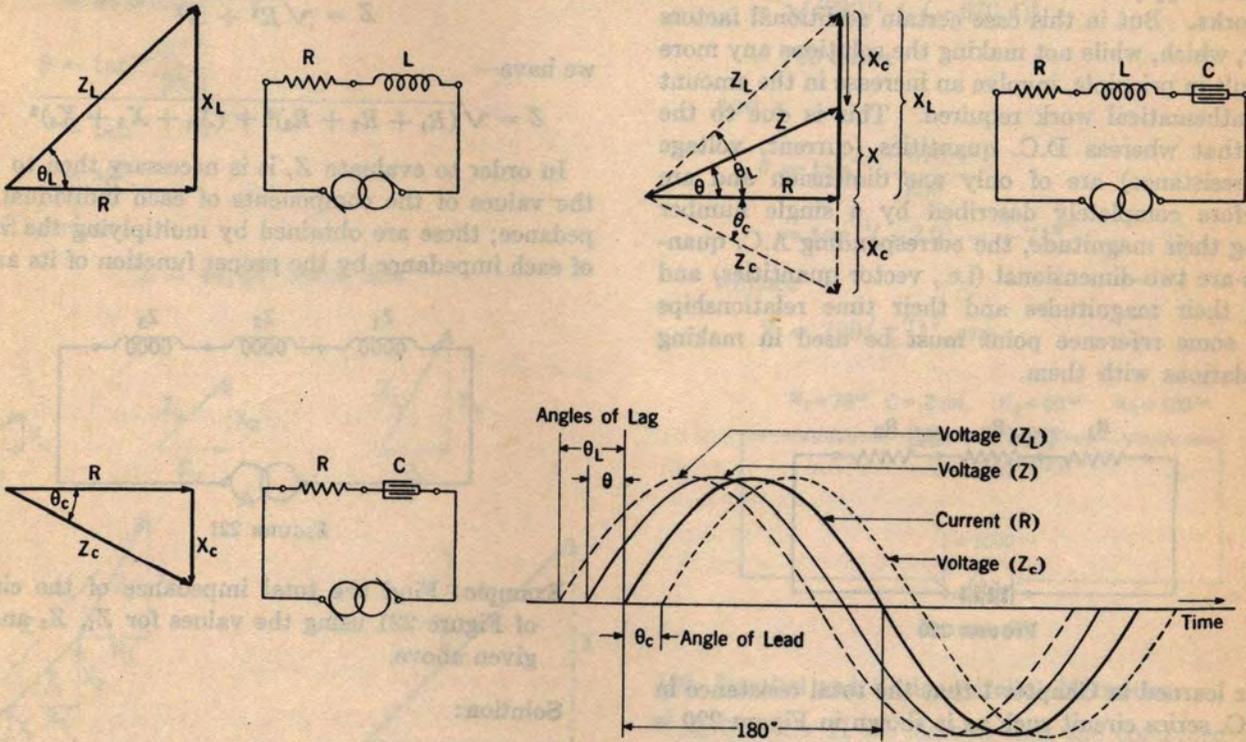


FIGURE 219

pletely describe it, however, since there could be any number of resistance, capacity and inductance combinations which would give the same numerical value. It is essential, therefore, to include an additional factor which will indicate the relative magnitudes of the resistance and reactance components of the impedance, in order to completely define it. This factor is the angle shown as  $\theta$  in Figures 218 and 219. Impedance is customarily expressed, accordingly, in the form  $Z/\theta$  ( $Z$  at an angle  $\theta$ ) where  $Z$  is the magnitude of the

impedance and  $\theta$  is the angle of lag or lead between any E.M.F. impressed across the impedance and the resultant current. As may be seen from Figure 218,  $\theta$  is equal to  $\tan^{-1} \frac{X}{R}$  (the angle whose tangent is  $\frac{X}{R}$ ). Also, by simple trigonometry we know that  $R = Z \cos \theta$  and  $X = Z \sin \theta$ . Thus, with the impedance expressed in the form  $Z/\theta$  it is completely defined and we may readily determine the magnitude of its resistance and reactance components.

TABLE VIII  
CHART OF VECTOR RELATIONS



PROPERTY	REACTANCE	IMPEDANCE	PHASE ANGLE
Inductance (L).....	$X_L = 2\pi fL$	$Z_L = \sqrt{R^2 + X_L^2}$	$\theta_L = \tan^{-1} \frac{X_L}{R}$
Capacity (C).....	$X_C = -\frac{1,000,000}{2\pi fC}$	$Z_C = \sqrt{R^2 + X_C^2}$	$\theta_C = \tan^{-1} \frac{X_C}{R}$
Net Effect.....	$X = X_L + X_C$	$Z = \sqrt{R^2 + X^2}$	$\theta = \tan^{-1} \frac{X}{R}$

- Notes: 1. If lines  $Z_C$ ,  $Z_L$  or  $Z$  represent phase of voltage, line  $R$  will indicate lead or lag of current and  $\theta_C$ ,  $\theta_L$  and  $\theta$  will be angle of lead or lag  
 2. Power factor is cosine of phase angle (Power =  $EI \cos \theta$ ).  
 3. The impedance symbol is usually written  $Z/\theta$ , for example,  $Z/\theta = 15^\circ/30^\circ$ , etc.

## CHAPTER XVII

### ALTERNATING CURRENTS—(Continued)

#### 108. Series Networks

In Chapters I and II, means of solving direct-current networks for the current values in the various branches were described. The same methods and formulas apply to the solution of alternating-current networks. But in this case certain additional factors enter, which, while not making the solutions any more difficult in principle, involve an increase in the amount of mathematical work required. This is due to the fact that whereas D.C. quantities (current, voltage and resistance) are of only one dimension and are therefore completely described by a single number giving their magnitude, the corresponding A.C. quantities are two-dimensional (i.e., vector quantities) and both their magnitudes and their time relationships with some reference point must be used in making calculations with them.

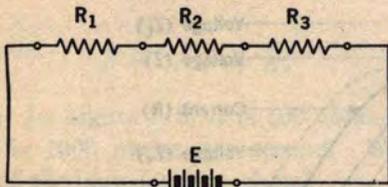


FIGURE 220

We learned in Chapter I that the total resistance in a D.C. series circuit such as is shown in Figure 220 is equal to the arithmetic sum of the individual resistances, or—

$$R = R_1 + R_2 + R_3, \text{ etc.} \quad (4)$$

Similarly in an A.C. series circuit, as shown in Figure 221, the total impedance is equal to the vector sum of the individual impedances or—

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3 \quad (61)$$

the bars over the impedance symbols meaning that they are vectors and to be treated accordingly in performing the indicated additions.

To graphically illustrate the application, let us assume that  $Z_1 = 10$  ohms with  $\theta_1 = 30^\circ$ ,  $Z_2 = 15$  ohms with  $\theta_2 = 45^\circ$  and  $Z_3 = 20$  ohms with  $\theta_3 = 60^\circ$ ; we then have the three vectors represented by Figure 222-A which, when added, give the value of  $Z$  shown in Figure 222-B. If we should represent not only the impedance vectors but the resistance and reactance

components as well, we should find that each group-of components adds algebraically as shown by Figure 223. By comparing Figure 223-C with Figure 223-B, we find that  $X$  is the sum of  $X_1$ ,  $X_2$  and  $X_3$  and  $R$  is the sum of  $R_1$ ,  $R_2$  and  $R_3$ . Therefore since—

$$Z = \sqrt{R^2 + X^2}$$

we have—

$$Z = \sqrt{(R_1 + R_2 + R_3)^2 + (X_1 + X_2 + X_3)^2} \quad (62)$$

In order to evaluate  $Z$ , it is necessary then to find the values of the components of each individual impedance; these are obtained by multiplying the value of each impedance by the proper function of its angle.

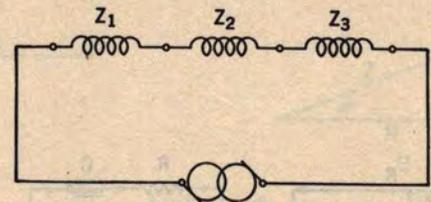


FIGURE 221

**Example:** Find the total impedance of the circuit of Figure 221 using the values for  $Z_1$ ,  $Z_2$  and  $Z_3$  given above.

**Solution:**

$$X = Z \sin \theta$$

$$R = Z \cos \theta$$

This gives

$$X_1 = 10 \times .500 = 5 \text{ ohms}$$

$$R_1 = 10 \times .866 = 8.7 \text{ ohms}$$

The other values can be determined in the same

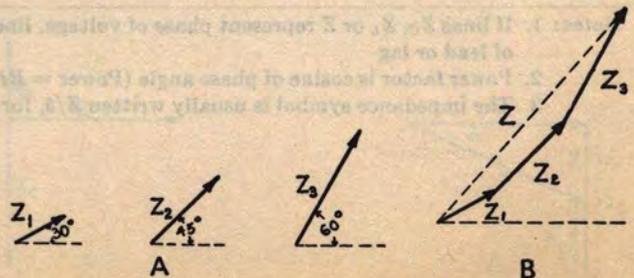


FIG. 222. GRAPHICAL ADDITION OF IMPEDANCE VECTORS

way, and we find that—

$$X_2 = 10.6 \text{ ohms}$$

$$R_2 = 10.6 \text{ ohms}$$

$$X_3 = 17.3 \text{ ohms}$$

$$R_3 = 10.0 \text{ ohms}$$

Applying Equation (62)—

$$\begin{aligned} Z &= \sqrt{(R_1 + R_2 + R_3)^2 + (X_1 + X_2 + X_3)^2} \\ &= \sqrt{(8.7 + 10.6 + 10)^2 + (5 + 10.6 + 17.3)^2} \\ &= \sqrt{(29.3)^2 + (32.9)^2} \\ &= 44.0 \text{ ohms} \end{aligned}$$

$$\theta = \tan^{-1} \frac{32.9}{29.3}$$

$$= \tan^{-1} 1.12$$

$$= 48^\circ.$$

Therefore

$$Z = 44/48^\circ \text{ ohms, ans.}$$

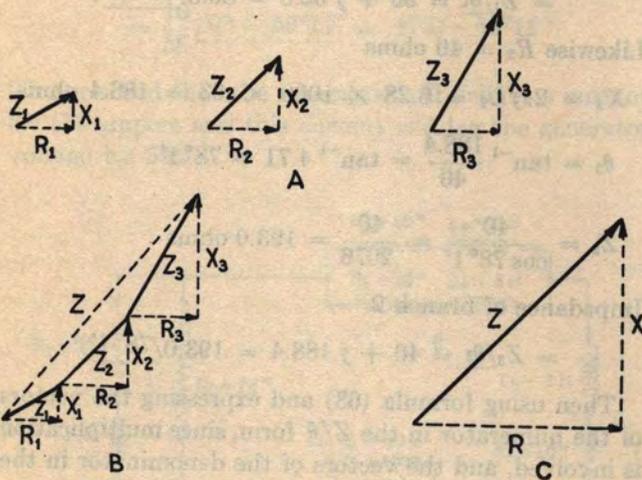


FIG. 223. ANALYSIS OF IMPEDANCE VECTOR ADDITION

The foregoing calculation covers a general case. In practice, however, we usually have given the inductance, capacity and resistance values rather than the individual impedances with their respective angles.

**Example:** Find the impedance of the series circuit shown by Figure 224.

**Solution:**

$$Z = \sqrt{(R_1 + R_2 + R_3)^2 + (X_c + X_L)^2}$$

where

$$X_c = -\frac{1,000,000}{2\pi f C}$$

$$= -\frac{1,000,000}{6.28 \times 1000 \times .2}$$

$$= -796 \text{ ohms}$$

and

$$X_L = 2\pi f L$$

$$= 6.28 \times 1000 \times .02$$

$$= 125.6 \text{ ohms.}$$

Then

$$\begin{aligned} Z &= \sqrt{(70 + 60 + 100)^2 + (-796 + 125.6)^2} \\ &= \sqrt{(230)^2 + (-670.4)^2} \\ &= 709 \text{ ohms} \end{aligned}$$

and

$$\theta = \tan^{-1} \frac{-670.4}{230}$$

$$= \tan^{-1}(-2.9) = -71^\circ,$$

whence

$$Z = 709/-71^\circ, \text{ ans.}$$

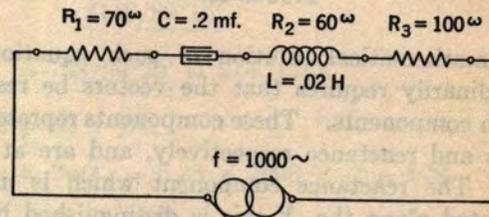


FIGURE 224

### 109. Parallel and Series-parallel Networks

In Chapter II we learned that the combined resistance of two parallel resistances was equal to—

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad (8)$$

or that if more than two resistances are in parallel, the combined resistance may be found by adding together the reciprocals of each resistance (called conductance) and taking the reciprocal of this value. That is—

$$G = G_1 + G_2 + G_3 \quad (10)$$

or

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Now if we substitute impedance for resistance in the above equations, they will hold for the A.C. case, providing that we remember that impedances are

vector quantities. Thus for two impedances in parallel, we may write the value of the combined impedance as—

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} \quad (63)$$

or, for more than two in parallel,

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}, \text{ etc.} \quad (64)$$

which latter may also be written—

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3, \text{ etc.} \quad (65)$$

where  $Y$  represents the reciprocal of impedance and is called **admittance**.

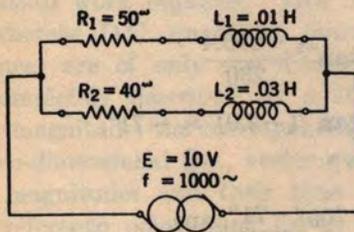


FIGURE 225

The mathematical solution of such equations as (63) ordinarily requires that the vectors be resolved into two components. These components represent resistance and reactance respectively, and are at right angles. The reactance component which is usually represented along the  $Y$ -axis is distinguished by the coefficient "j", to indicate its position relative to the resistance component along the  $X$ -axis. The vector is then expressed in the standard notation as  $\bar{Z} = Z/\theta = R + jX$  where "j" indicates a rotation of  $90^\circ$  in a counterclockwise direction. In the algebra of complex quantities, "j" is then handled like any ordinary coefficient. The use of this notation makes possible the direct application of the same formulas as those used in D.C. calculations to the solution of A.C. networks. As an example, let us determine the current delivered by the generator of Figure 225 and the phase angle of this current with the generator E.M.F.

By Ohm's Law we know that the total current delivered by the generator is—

$$I = \frac{E}{Z/\theta}$$

where  $Z/\theta$  is the total impedance of the circuit and consists of the net impedance of the two parallel paths whose individual impedances may be indicated as  $Z_1/\theta_1$  and  $Z_2/\theta_2$ . Then from the usual formula for parallel circuits—

$$Z/\theta = \frac{Z_1/\theta_1 \times Z_2/\theta_2}{Z_1/\theta_1 + Z_2/\theta_2} \quad (63)$$

The first step is to find the values of  $Z_1/\theta_1$  and  $Z_2/\theta_2$ . We know that—

$$\theta_1 = \tan^{-1} \frac{X_1}{R_1}$$

where

$$R_1 = 50 \text{ ohms}$$

and

$$X_1 = 2\pi f L_1 = 6.28 \times 1000 \times .01 = 62.8 \text{ ohms}$$

Then

$$\theta_1 = \tan^{-1} \frac{62.8}{50} = \tan^{-1} 1.255 = 51^\circ 27'$$

Then, since  $R_1 = Z_1 \cos \theta_1$ —

$$Z_1 = \frac{R_1}{\cos \theta_1} = \frac{50}{\cos 51^\circ 27'} = \frac{50}{.6232} = 80.3 \text{ ohms}$$

Impedance of branch 1

$$= Z_1/\theta_1 = 50 + j 62.8 = 80.3/51^\circ 27'$$

Likewise  $R_2 = 40$  ohms

$$X_2 = 2\pi f L_2 = 6.28 \times 1000 \times .03 = 188.4 \text{ ohms}$$

$$\theta_2 = \tan^{-1} \frac{188.4}{40} = \tan^{-1} 4.71 = 78^\circ 1'$$

$$Z_2 = \frac{40}{\cos 78^\circ 1'} = \frac{40}{.2076} = 193.0 \text{ ohms}$$

Impedance of branch 2

$$= Z_2/\theta_2 = 40 + j 188.4 = 193.0/78^\circ 1'$$

Then using formula (63) and expressing the vectors of the numerator in the  $Z/\theta$  form, since multiplication is involved, and the vectors of the denominator in the  $R + jX$  form, since addition is involved, we have—

$$\begin{aligned} Z/\theta &= \frac{Z_1/\theta_1 \times Z_2/\theta_2}{(R_1 + jX_1) + (R_2 + jX_2)} \\ &= \frac{Z_1 Z_2 / \theta_1 + \theta_2}{(R_1 + R_2) + j(X_1 + X_2)} \\ &= \frac{80.3 \times 193.0 / 51^\circ 27' + 78^\circ 1'}{(50 + 40) + j(62.8 + 188.4)} \\ &= \frac{15500 / 129^\circ 28'}{90 + j251.2} \\ &= \frac{15500 / 129^\circ 28'}{90} \bigg/ \frac{\tan^{-1} 251.2}{90} \end{aligned}$$

$$\begin{aligned}
 &= \frac{15500/129^\circ 28'}{90} \\
 &= \frac{15500/129^\circ 28'}{\cos(\tan^{-1} 2.79) / \tan^{-1} 2.79} \\
 &= \frac{15500/129^\circ 28'}{90} \\
 &= \frac{15500/129^\circ 28'}{\cos 70^\circ 17' / 70^\circ 17'} \\
 &= \frac{15500/129^\circ 28'}{.3374 / 70^\circ 17'} \\
 &= \frac{15500/129^\circ 28'}{267 / 70^\circ 17'} \\
 &= \frac{15500}{267} / 129^\circ 28' - 70^\circ 17' \\
 &= 58.0 / 59^\circ 11'
 \end{aligned}$$

And

$$\begin{aligned}
 I &= \frac{E}{Z/\theta} = \frac{10}{58/59^\circ 11'} \\
 &= \frac{10}{58} / 0^\circ - 59^\circ 11' = .173 / -59^\circ 11'
 \end{aligned}$$

Thus we find that the generator will deliver a current of .173 ampere and this current will lag the generator voltage by  $59^\circ 11'$ .

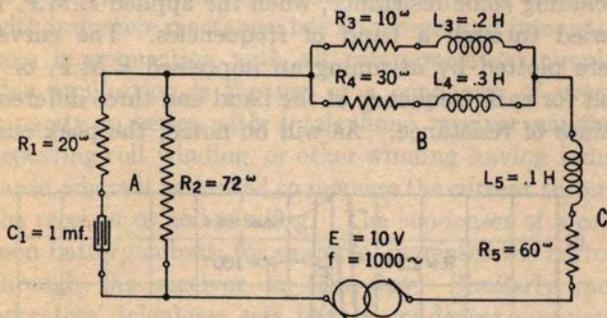


FIGURE 226

With a little practice it will be found that several of the detailed steps given above can be performed in a single operation. This may be illustrated by solving the circuit of Figure 226 to find the current delivered by the generator and its phase relationship with the E.M.F.

**Solution:**

$$Z_1 = R_1 + jX_1$$

$$X_1 = -\frac{10^6}{2\pi f C_1} = -\frac{1,000,000}{6.28 \times 1000 \times 1} = -159.3$$

$$Z_1 = 20 - j159.3 = 160.7 / -82^\circ 51'$$

$$Z_2 = R_2 + jX_2 = 72 + j0 = 72 / 0^\circ$$

$$Z_A = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{160.7 / -82^\circ 51' \times 72 / 0^\circ}{20 - j159.3 + 72 + j0}$$

$$= \frac{11,570 / -82^\circ 51'}{92 - j159.3} = \frac{11,570 / -82^\circ 51'}{184 / -60^\circ 1'}$$

$$= 62.8 / -22^\circ 50' = 58.0 - j24.4$$

$$Z_3 = R_3 + jX_3$$

$$X_3 = 2\pi f L_3 = 6.28 \times 1000 \times .2 = 1256$$

$$Z_3 = 10 + j1256 = 1256 / 89^\circ 33'$$

$$Z_4 = R_4 + jX_4$$

$$X_4 = 2\pi f L_4 = 6.28 \times 1000 \times .3 = 1884$$

$$Z_4 = 30 + j1884 = 1884 / 89^\circ 5'$$

$$Z_B = \frac{Z_3 Z_4}{Z_3 + Z_4} = \frac{1256 / 89^\circ 33' \times 1884 / 89^\circ 5'}{10 + j1256 + 30 + j1884}$$

$$= \frac{2,365,000 / 178^\circ 38'}{40 + j3140}$$

$$= \frac{2,365,000 / 178^\circ 38'}{3140 / 89^\circ 16'}$$

$$= 753 / 89^\circ 22' = 8 + j753$$

$$Z_C = R_5 + jX_5$$

$$X_5 = 2\pi f L_5 = 6.28 \times 1000 \times .1 = 628$$

$$Z_C = 60 + j628$$

The total impedance  $Z = Z_A + Z_B + Z_C$

$$\begin{aligned}
 Z &= 58.0 - j24.4 + 8 + j753 + 60 + j628 \\
 &= 126 + j1356 = 1360 / 84^\circ 40'
 \end{aligned}$$

$$I = \frac{E}{Z} = \frac{10}{1360 / 84^\circ 40'} = .00735 / -84^\circ 40'$$

The current delivered by the generator has a value of .00735 ampere and lags the impressed voltage by  $84^\circ 40'$ .

## 110. Alternating-Current Resistance

In alternating-current networks, the apparent resistance of a particular piece of apparatus is often quite different from its direct current or true resistance. As shown by Table V (Chapter VIII), the resistance offered to alternating current may be much greater than that offered to direct current; furthermore, in such cases the value of the resistance depends to some extent on the alternating-current frequency. We find, then, that **not only the reactance component of an**

impedance but its resistance component as well may be a function of the frequency.

"Alternating-current resistance", so called to distinguish it from direct current or true resistance, represents not only the actual resistance of the conductor used to wind a coil but includes also a factor due to the power losses within the iron core. That is to say, when a current flows through a coil winding and establishes a strong magnetic field in the core first in one direction and then in the other, there are certain power losses within the iron due to a heating effect. This is caused in part by hysteresis and in part by small currents induced in the iron itself as a conductor, and called "eddy currents". The total power loss in the coil includes not only the heat losses due to the resistance of the coil winding but also the core losses. Since any power loss can be expressed in the form of the Equation  $P = I^2R$ , we assume that the winding has in effect a resistance which satisfies this equation. But it so happens that the part of the power loss that is due to the iron core increases with the frequency. Therefore, we should expect the A.C. resistance for a high frequency to be greater than the A.C. resistance for a low frequency.

### 111. Resonance

In a circuit containing a given inductance, the reactance,  $X_L$ , depends upon the frequency; if the frequency is doubled, the reactance is also doubled. In the case of a given capacity value, on the other hand, the negative reactance,  $X_c$ , is reduced one-half by doubling the frequency. If a series circuit contains both inductance and capacity, as shown in Figure 227-A, there is therefore some frequency at which the negative reactance,  $X_c$ , becomes equal but opposite in value to  $X_L$ . The combined reactance is then equal to zero, as shown in Figure 227-B where the dotted line crosses the zero axis. This is called the frequency to which the circuit is resonant, or where—

$$0 = 2\pi fL - \frac{1,000,000}{2\pi fC}$$

The value of the resonant frequency,  $f_r$ , can be determined in terms of the inductance and capacity by solving this equation for  $f$  as follows:

$$f_r = \frac{1,000}{2\pi\sqrt{LC}} \quad (66)$$

Since the total reactance is zero at the resonant frequency, the impedance is then equal to the resistance of the circuit and the current flow is determined solely by this resistance value.

Figure 228 illustrates the behavior of a series resonant circuit similar to that shown in Figure 227-A, but

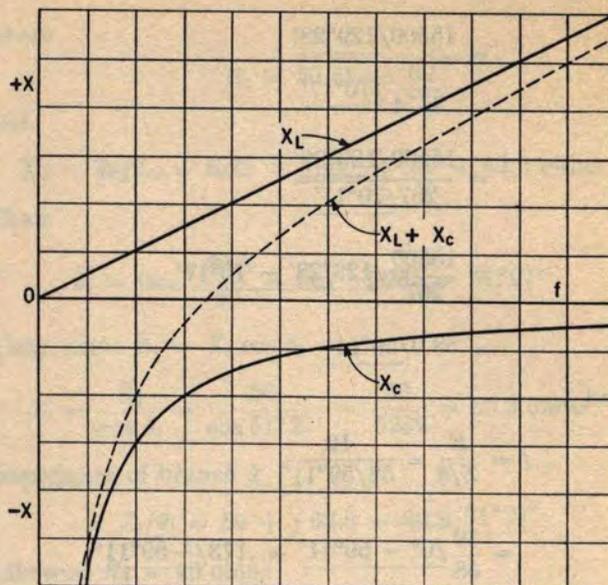
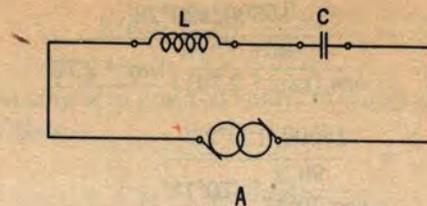


FIG. 227. SERIES RESONANT CIRCUIT

including some resistance, when the applied E.M.F. is varied through a band of frequencies. The curves were plotted by assuming an impressed E.M.F. of 1 volt for each frequency of the band and three different values of resistance. As will be noted, the peak cur-

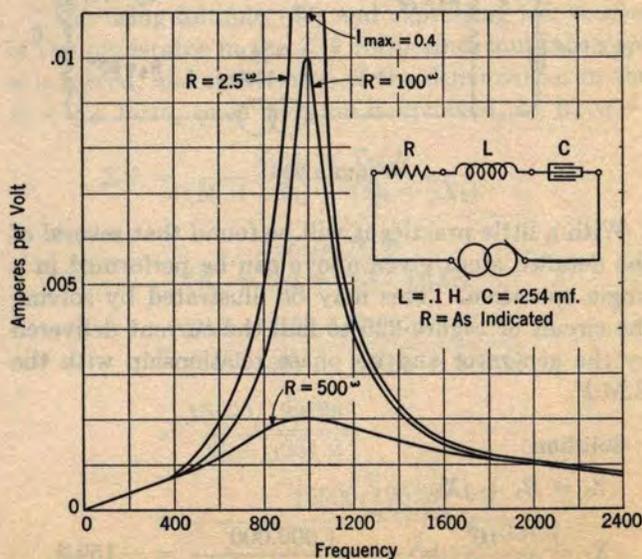


FIG. 228. CURVES OF CURRENT VALUES IN SERIES RESONANT CIRCUIT

rent values depend entirely upon the resistance values, for at the peak the positive and negative reactances exactly neutralize each other and the current is determined solely by the resistance. Accordingly, the addition of resistance to the series resonant circuit reduces the selectivity or sharpness of the resonance peak. That is, the ratio of the current at the resonant frequency to the current at frequencies near the resonant frequency is reduced.

**Example:** To what frequency is the circuit shown by Figure 228 resonant if  $C$  is .254 mf.,  $L$  is .10 H, and what current will flow at resonance when  $R$  is 4 ohms and  $E$  is 1.0 volt?

**Solution:**

$$f_r = \frac{1,000}{6.28 \sqrt{.10 \times .254}}$$

$$= \frac{1,000}{6.28 \sqrt{.0254}}$$

$$= \frac{1,000}{6.28 \times .159}$$

$$= 1,000 \text{ cycles per sec. Ans.}$$

$$I = \frac{E}{R} = \frac{1.0}{4} = .25 \text{ amp. Ans.}$$

The resonance principle in its broadest applications, or rather the practice of neutralizing capacity reactance with inductive reactance, has numerous and interesting uses in connection with all communication circuits. One application is the use of a condenser of proper capacity in series with a telephone receiver winding, repeating coil winding, or other winding having inductance, where it is desired to increase the current through the receiver or coil winding. The condenser of a common battery subset, for example, increases the current through the receiver in this way. Similarly most operators' telephone sets have a condenser associated with the induction coil.

A second application of the resonance principle is the so-called "tuned" circuit, or the resonant circuit used for selectivity. It is an arrangement whereby the circuit has a much lower impedance to some particular frequency than to any other frequency; if a band of frequencies is impressed, it selects, so to speak, a high current for the particular frequency but permits only a negligible current for any other frequency. Figure 228 illustrates this principle.

Another connection of the tuned circuit, shown in Figure 229-A, is called the **anti-resonant** connection. For this condition, when the positive reactance is equal and opposite to the negative reactance, the combined impedance presented to the generator is ex-

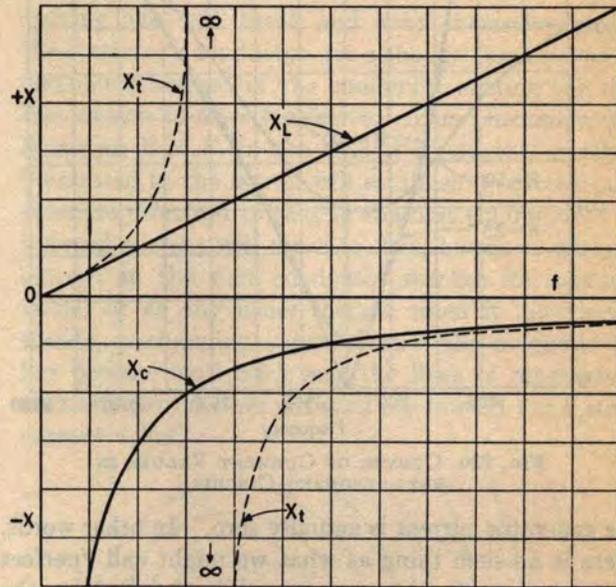
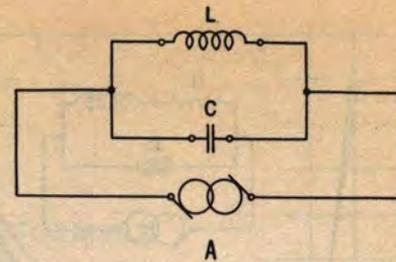


FIG. 229. ANTI-RESONANT CIRCUIT

tremely great and there is a minimum load on the generator. In other words, the generator circuit is practically open. Figure 229-B shows the combined reactance,  $X_t$ , presented to the generator by this circuit. It can be seen that at the resonant frequency the two parallel reactances combine to give an extremely high value. At the same time, there must be a current through the inductance, determined by dividing the voltage of the generator by the impedance of this branch. Similarly, there must be a current through the condenser which can be determined in the same way. These currents are equal in value, but are flowing in opposite directions, thereby neutralizing each other in the lead to the generator. Effectively, this gives an open circuit in so far as the generator is concerned, but a circuit equal to either the inductance or capacity alone connected to the generator in so far as either of the branches is concerned. The physical explanation here is that a current is oscillating around through the inductance and condenser, with the E.M.F. of the generator merely sustaining this oscillation. Of course, since the inductance must have some resistance, there will be an  $I^2R$  loss in the inductance, and it would never be possible to have the theoretical case where

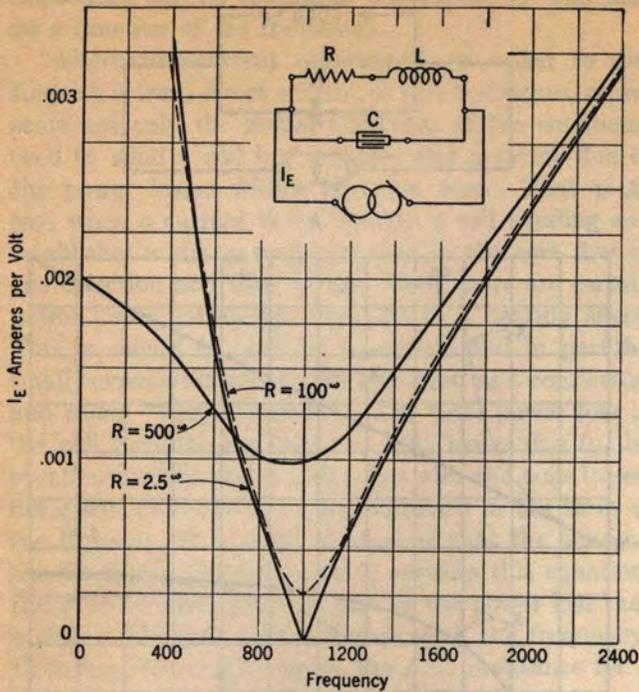


FIG. 230. CURVES OF CURRENT VALUES IN ANTI-RESONANT CIRCUIT

the generator current is actually zero. In other words, there is no such thing as what we might call "perfect anti-resonance", where no energy is supplied by the generator and a current is maintained indefinitely by energy oscillating back and forth from the condenser to the coil.

Figure 230 illustrates the selectivity of an anti-resonant circuit made up of the same units as were used in the series resonant circuit. It will be noted that the selectivity of the anti-resonant circuit is also decreased as the resistance is increased. Indeed, there is a value of resistance beyond which the circuit loses its resonant characteristics altogether. Moreover, in this case, the resistance may be seen to have some effect on the value of the resonant frequency.

Perhaps the most widely known application of both resonant and anti-resonant circuits is in connection with radio sending and receiving sets. From our view-

point, the most interesting, perhaps, is the carrier application which is discussed in a later chapter.

Numerous other applications are possible. Thus a simple series resonant circuit may be used as a substitute for a step-up transformer, where an E.M.F. greater than the impressed E.M.F. but with little current drain, is desired. An example of such use may be found in certain vacuum tube circuits, where the operation depends upon the value of the impressed E.M.F. on the grid (which is practically an open circuit) rather than upon the current strength of the incoming energy.

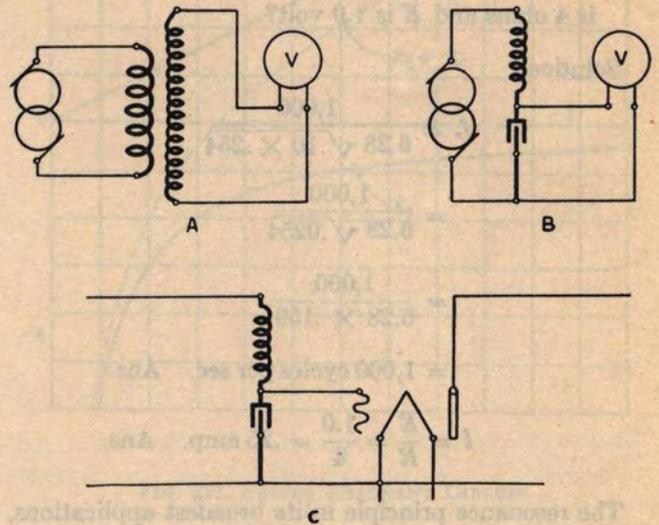


FIG. 231. USE OF RESONANCE PRINCIPLE TO INCREASE VOLTAGE

Figure 231 illustrates this principle. Here Sketch A illustrates the step-up transformer while Sketch B shows how a resonant circuit can be used to greatly increase the E.M.F. of the generator at a single frequency, thereby accomplishing the same result as the transformer. If a voltmeter is connected across the condenser alone, it will be found that the voltage is many times that of the generator at the resonant frequency. Sketch C illustrates the connection of the grid circuit of a vacuum tube for securing a higher potential than that which is impressed from the line.

## CHAPTER XVIII

### REPEATING COILS AND TRANSFORMERS

#### 112. Mutual Induction

The inductive effects discussed in Chapter VIII dealt with the magnetic interlinkages from one turn of a coil winding to the other turns of the same winding. We defined the effects coming from such magnetic interlinkages as "self-inductance". The current resulting from the induced E.M.F. was superposed upon the current resulting from the impressed E.M.F.

In practice, we may experience inductive effects in circuits other than the one in which the current due to the impressed E.M.F. is flowing. That is to say, two coils may be so related that the lines of magnetic induction established by a current in the first coil may cut the turns of the second coil (which may be connected to an entirely different circuit) in the same way that similar lines established by any one turn of a single coil cut the other turns of the same coil. This effect is called **mutual induction** and the property of the electrical circuit that is responsible for the effect is known as its **mutual inductance**.

#### 113. Theory of the Transformer

In the study of magnetism we found that a wire in which there is a current is always surrounded by a magnetic field. This field, when created by a current establishing itself in the conductor, grows outwardly from the wire as the current increases. Figure 232

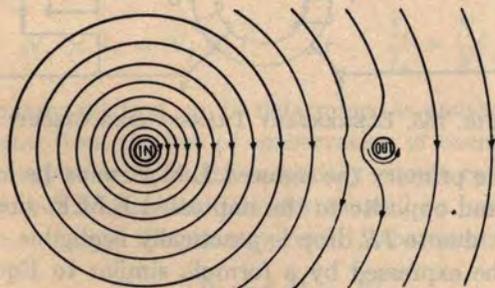


FIGURE 232

shows a group of lines of magnetic induction around a conductor (shown in cross-section) in which the current is increasing in value. If a second conductor is in the vicinity, it will be cut by these lines moving outward from the current-carrying conductor. In the same manner that stationary lines seem to break and wrap themselves around a moving conductor (Figure 70), the

moving lines will break and wrap themselves around the stationary conductor, for although the lines cut the conductor instead of the conductor cutting the lines, the motion is merely relative. This phenomenon induces an E.M.F. in the second conductor, which, as illustrated in the figure, will establish a current in the opposite direction to that in the first conductor. The induced current will cease to flow, however, when the current in the first conductor reaches its maximum value, or at any other instant when it may have a steady, unchanging value because the magnetic field has become stationary and the lines of magnetic induction move neither outward nor inward for a steady current value.

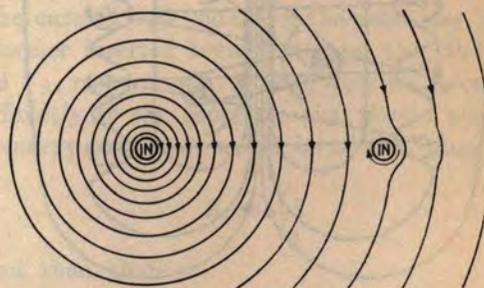


FIGURE 233

If the current in the first conductor is decreased, we have the reverse condition, or that shown in Figure 233. Here the lines, instead of expanding and moving outward, are contracting and moving inward, cutting the second conductor as formerly, but now the current induced is in the opposite direction. It is now in the same direction in the second conductor as in the first. This law for induced E.M.F. may be expressed as follows: **For any two parallel conductors, a current in one increasing in value induces an E.M.F. in the other, tending to establish a current in the opposite direction, and a current decreasing in one will induce an E.M.F. in the other, tending to produce a current in the same direction.**

Instead of the two single conductors shown in Figures 232 and 233, let us consider two separate coils, one inside the other, as in Figure 234. If we call the one carrying the original current the "primary", which in this case we may represent by the inside coil, and the other the "secondary", we shall find that a strong magnetic field is established by a changing current in

the primary. This will cut the entire group of conductors represented by the turns of the secondary, thereby inducing appreciable potential in the secondary. The ordinary telephone induction coil operates in this manner. The primary, when connected in series with the transmitter, carries a current which decreases and increases in value in response to the varying resistance of the transmitter. Consequently, an alternating current is induced in the secondary of the coil.

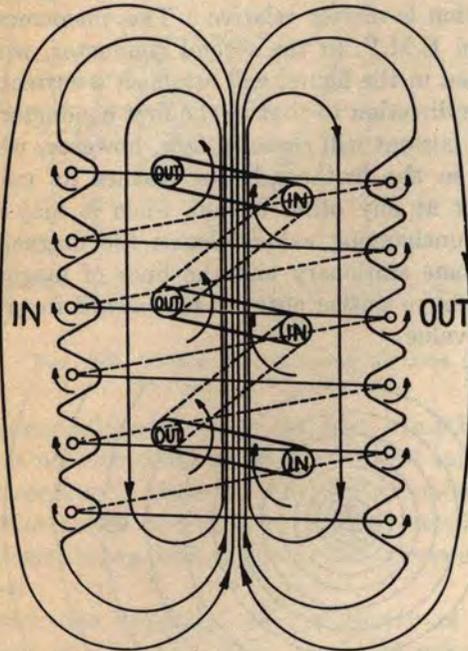


FIG. 234. PRINCIPLE OF INDUCTION COIL

If now the two separate coils of Figure 234 are wound on the same iron core in the manner indicated by Figure 235, the effect will be intensified. Because the iron offers a path of low reluctance to the magnetic flux, the total number of lines will be greatly increased and all of the lines set up by the primary winding, *P*, will cut all of the secondary winding, *S*.

If the windings, *P* and *S* have the same number of turns, and both the coils and core are constructed so as to have negligible energy losses, we shall find that the voltmeter reading is the same when connected across the terminals of *S* as when connected across the terminals of *P*. In other words, the induced E.M.F. of the secondary winding is equal to the impressed E.M.F. of the primary winding. Such a device is called an ideal transformer of unity ratio.

If, now, we should increase the number of turns of the secondary winding *S*, we would find that the voltmeter reading would be greater on the secondary than on the primary side of the transformer. If we should decrease the number of turns of the winding *S*, the effect

would be reversed. We have here a means, therefore, of controlling the voltage applied to a load; we may effectively increase or decrease the generator voltage by a proper choice of transformer. If a transformer has a greater number of turns on the secondary than on the primary so that the voltage is increased, it is called a "step-up" transformer; if it has a lesser number of turns on the secondary than on the primary so that the voltage is decreased, it is called a "step-down" transformer. The voltage across the two windings is directly proportional to the number of turns. This relation is expressed by the equation:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \quad (67)$$

We may explain this relation between the number of turns and voltage by our original law governing inductive effects, which states that the induced voltage is proportional to the rate of cutting lines of magnetic induction. Each time the alternating E.M.F. in the primary completes a cycle, it establishes a magnetic flux in the iron core which collapses to be established in the opposite direction, to again collapse, etc. This flux must cut each and every turn about the iron core. In doing so, for the ideal case where there is no loss due to magnetic leakage, etc., the same voltage is induced in each individual turn. This voltage may be represented by the symbol *v*. Now, the voltage measured across the secondary (with no load connected) is merely the sum of these individual turn voltages or—

$$V_S' = N_S \times v \quad (68)$$

where *N<sub>s</sub>* is the number of turns on the secondary.

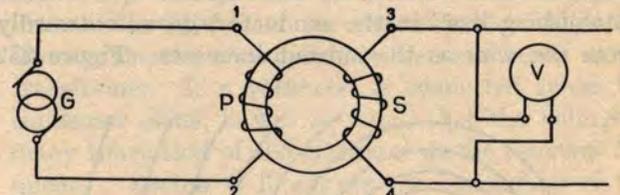


FIG. 235. ELEMENTARY TRANSFORMER CIRCUIT

In the primary the induced E.M.F. must be exactly equal and opposite to the impressed E.M.F. since the E.M.F. due to *IR* drop is practically negligible. This could be expressed by a formula similar to Equation (68), thus—

$$V_P = N_P \times v \quad (69)$$

Since *v* is the same in both Equations (68) and (69), we may derive Equation (67) by dividing (69) by (68).

In Figure 235 the current being supplied by the generator is negligible, inasmuch as we have considered the transformer as having no energy losses. If, however, a load in the form of a shunting impedance is

connected to its secondary as shown by Figure 236, the induced E.M.F. in the winding  $S$  causes a current to flow through the impedance  $Z_s$ , which from Equation (50), can be expressed as follows—

$$I_s = \frac{V_s}{Z_s}$$

When this current starts to flow through the load  $Z_s$ , and through the winding  $S$ , it will establish other lines of magnetic induction in the transformer core, which oppose those established by the current in the winding  $P$ . This will tend to neutralize the magnetic field in the iron core, thereby tending to counteract the inductance of the winding  $P$  and to make it more nearly like a plain resistance. With the induced E.M.F. in the winding  $P$  reduced, a greater current will flow from the generator through this winding, thus again increasing the flux in the iron core, so that finally there are produced the same induced E.M.F. effects as in the case of the transformer on open circuit. We therefore find that the transformer adjusts itself to any load that may be connected to the secondary just as if an equivalent load were connected directly to the generator, i.e., the current supplied by the generator increases with an increase of current in the secondary of the transformer.

This current, however, is not necessarily of the same value in the primary as in the secondary, but like the voltage, depends upon the ratio of the number of turns of the primary to the number of turns of the secondary. The relation between current values is the inverse ratio of the number of turns. In other words, the winding having the greater number of turns has a proportionately smaller current. This is seen when we consider that the flux in the core depends upon the current value times the number of turns, and the flux established by one coil balances that established by the other—

$$N_p \times I_p = N_s \times I_s \quad \text{or} \quad \frac{I_s}{I_p} = \frac{N_p}{N_s} \quad (70)$$

The same relation can be determined in another way. We know from the law of conservation of energy that the energy existing in the secondary circuit can never exceed, but for an ideal transformer will be just equal to, the energy of the primary circuit, where since—

$$P_p = P_s \text{ and } P = EI,$$

we have—

$$V_s I_s = V_p I_p$$

from which—

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} \quad \text{or} \quad \frac{I_s}{I_p} = \frac{N_p}{N_s} \quad (70)$$

Though we find that connecting the load  $Z_s$  to the

secondary of the transformer causes the generator to furnish a current output in much the same way as if a load were connected across the generator, it does not follow that the same current flows through the load  $Z_s$  with the transformer inserted between the generator and  $Z_s$  as would flow if  $Z_s$  were connected directly to the generator without the transformer. We have

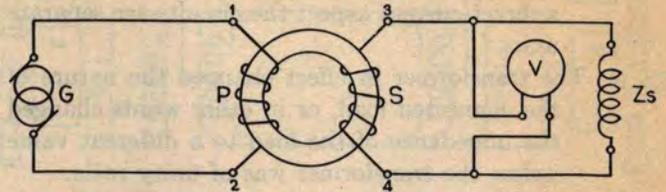


FIGURE 236

just seen that the voltages measured on the two sides of the transformer are directly proportional to the number of turns, and we know, moreover, from Equation (50) that—

$$Z_s = \frac{V_s}{I_s}$$

But the current and voltage of the generator with the transformer inserted between it and the load  $Z_s$  are  $I_p$  and  $V_p$ , respectively, so that were we to connect a load directly to the generator that would absorb the same energy output, it would be of the value—

$$Z_p = \frac{V_p}{I_p}$$

We find, then, that—

$$\frac{Z_p}{Z_s} = \frac{V_p}{I_p} \div \frac{V_s}{I_s} = \frac{V_p}{I_p} \times \frac{I_s}{V_s} = \frac{V_p}{V_s} \times \frac{I_s}{I_p} = \frac{N_p}{N_s} \times \frac{N_p}{N_s}$$

or—

$$\frac{Z_p}{Z_s} = \left[ \frac{N_p}{N_s} \right]^2 \quad (71)$$

Inequality ratio transformers may be rated either according to their voltage ratios, step-up or step-down as the case may be, or in accordance with their impedance ratios. In power work where transformers are primarily used to change the voltage of the system, the rating is on the voltage basis. In telephone work where inequality transformers are used in most cases primarily to match unequal impedances, as will be explained later, they are usually rated in accordance with their impedance ratios.

Before taking up specific uses of the transformer, let us review in general what its presence in Figure 236 has or may have accomplished:

- a. The characteristics of the electrical energy may have been changed, or we might say its state may have been "transformed", inasmuch as in

the primary circuit we may have had high current and low voltage, while in the secondary circuit we may have had low current and high voltage, or vice versa, depending upon whether the transformer was step-up or step-down.

- b. The electrical energy was transferred from one circuit to another without any metallic connection being made between the two circuits; from a direct-current aspect the circuits are separate units.
- c. The transformer in effect changed the nature of the connected load, or in other words changed the impedance of the load to a different value unless the transformer was of unity ratio.

In power work the principal use of a transformer is to accomplish the result given in *a* above, whereas in telephone work we are directly concerned with *b* and *c*.

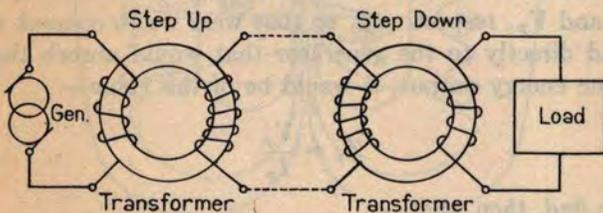
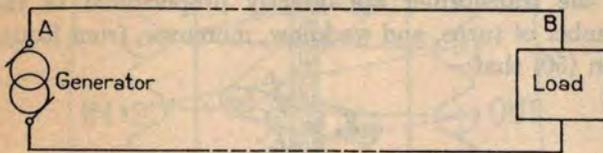


FIG. 237. USE OF TRANSFORMERS IN POWER CIRCUIT

First, let us illustrate the power case by referring to Figure 237 which shows the use of transformers in a simple power transmission circuit. Let us assume that a 110-volt alternating-current generator at station *A* is to be used to supply a load several miles away. The load is of such nature that it must have 100 amperes at an impressed voltage of 100 volts. Transmission from *A* to *B* must, therefore, be accomplished with a loss of 10 volts for a current of 100 amperes and this means that the  $IR$  drop of the line must not exceed 10 volts. Therefore, the resistance of the line, from the equation—

$$R = \frac{E}{I} = \frac{10}{100} = \frac{1}{10} \text{ ohm}$$

must not exceed 1/10th of an ohm, requiring extremely large copper conductors. If, however, a step-up transformer of 1-to-20 voltage ratio is inserted at the generator, and a step-down transformer of 20-to-1 ratio is inserted at the load, we shall find from the relation between current, voltage and power, that the current in

the transmission line will be equal to 5 amperes instead of 100 amperes. It will then be possible to have a 200 volt drop in the line and still have a voltage of 2000 on the primary of the transformer at the distant end, or the required 100 volts when stepped down. Since the current in the line will now be 1/20th of 100, or 5 amperes, the resistance of the line in this case will be—

$$R = \frac{200}{5} = 40 \text{ ohms}$$

We find, then, that the size of the conductors for the transmission line where the transformers are used, must be such that the resistance will not exceed 40 ohms, whereas in the first case it must be such that the resistance will not exceed 1/10th ohm. The amount of copper required in the second case is 1/400th or only 1/4th of one per cent of that required in the first case. The economy due to the copper saving is apparent.

#### 114. Transformer Applications to Telephone Circuits

The applications of transformers to telephone circuits are numerous and varied. The reduction of energy losses in alternating-current transmission, as illustrated in Figure 237, has an application to telephone transmission but is not so important as other uses. One very general use is to accomplish the result given as *b* above. In this case, the primary function of the coil is to transfer energy to another circuit rather than to change the voltage and current values. When so used in telephone work, they are generally called "repeating coils" rather than "transformers" because their primary function is to "repeat" the energy into a different primary circuit rather than to transform it into a different state. There are, however, inequality ratio repeating coils which perform both functions. On the other hand, in connection with telephone repeater circuits and certain other telephone apparatus, input and output coils are used primarily to change the voltage and current characteristics. These are accordingly called "transformers", and not "repeating coils".

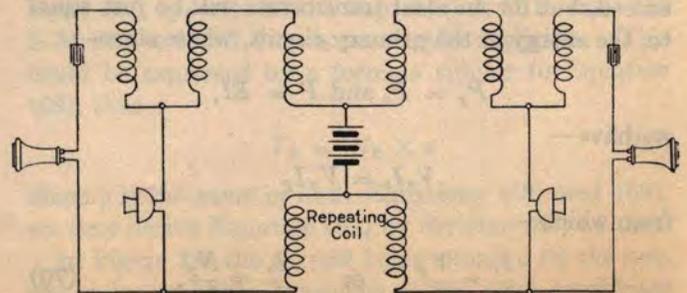


FIG. 238. TRANSFORMERS IN COMMON BATTERY TELEPHONE CONNECTION

Perhaps the most common application of the repeating coil in telephone work is in connection with the common battery cord circuit, as illustrated by Figure 238. Here the alternating-current flow in one subscriber's line is repeated into the other subscriber's line with little energy loss, and at the same time the windings of the coils afford the proper direct-current connections for each subscriber's station to receive a superposed D.C. current for transmitter supply. A similar use of the same type of coil in connection with a toll switching trunk circuit is illustrated in Figure 140. Here only one side of the coil is used for battery supply while a condenser is bridged at the mid-point of the other winding, which prohibits the flow of direct current from that side of the circuit. Here again the repeating coil accomplishes the transmission of voice current from one side to the other without its being appreciably affected by the direct-current features of the circuit.

Another very general use of repeating coils in the telephone plant is for deriving "phantom" circuits. Here the coils serve a unique purpose which has no counterpart in electrical power work, and is not included in the classification of transformer functions given above. We shall therefore need to consider this application more fully. However, it may be noted that the coils, while serving this particular purpose, may also function effectively as impedance matching devices.

### 115. The Phantom Circuit

Figure 239 is a simplified diagram of two adjacent and similar telephone circuits arranged for phantom operation. By means of repeating coils installed at the terminals of the wire circuits, a third telephone circuit is obtained. This third circuit is known as the phantom and utilizes the two conductors of each of the two principal, or "side" circuits, as one conductor of the third circuit. The two side circuits and the phantom circuit are together known as a phantom group. These three circuits, employing only four line conductors, can be used simultaneously without interference with each other, or without crosstalk between any combination, provided the four wires have identical electrical characteristics and are properly transposed to prevent crosstalk.

The repeating coils employed at the terminals are designed for voice-current and ringing-current frequencies, and do not appreciably impair transmission over the principal or side circuits. The third or phantom circuit is formed by connecting to the middle points of the line sides of the repeating coil windings, as shown in the figure. Since the two wires of each side circuit are identical, any current set up in the

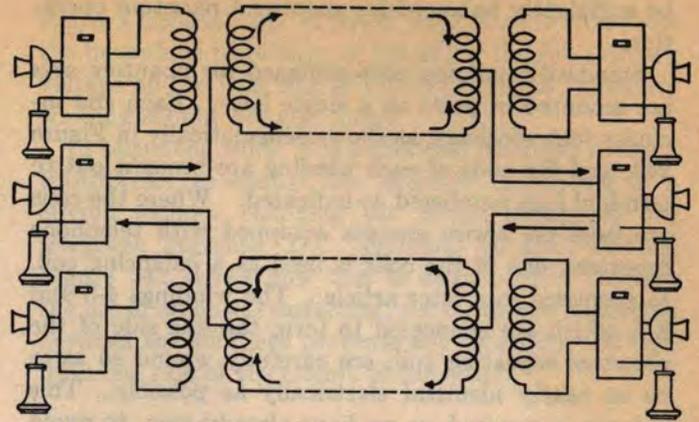


FIG. 239. PRINCIPLE OF THE PHANTOM CIRCUIT

phantom circuit will divide equally at the mid-point of the repeating coil line windings. One part of the current will flow through one-half of the line winding, and the other part of the current will flow in the opposite direction through the other half of the line winding. The inductive effects will be neutralized, and there will be no resultant current set up in the drop or switchboard side of the repeating coil. Since the phantom current divides into two equal parts, the halves will flow in the same direction through the respective conductors of one side circuit, and likewise return in the other side circuit. At any one point along a side circuit, there will be no difference of potential between the two wires due to current in the phantom circuit, and a telephone receiver bridged across them will not detect the phantom conversation.

Since there is no connection, inductive or otherwise, between the two circuits at the terminals, it is equally true that a conversation over a side circuit cannot be heard in the phantom. This can be understood by imagining a flow in the closed side circuit through the line wires and the windings of the repeating coils at each end. With the side circuit conductors electrically equal, there can be no difference of potential between the mid-point of the repeating coil line winding at one end and the mid-point of the repeating coil line winding at the other end because the drops of potential for the two parts of the side circuit are equal and opposite. If the side circuit, therefore, impresses no difference of potential on any part of the phantom circuit, the side circuit conversation cannot be heard over the phantom.

In the theory of the phantom it should not be forgotten that the conductors are assumed to be electrically identical, or in other words, the conductors are perfectly "balanced". The phantom is very sensitive to the slightest upset of this balance, and circuits that are sufficiently balanced to prevent objectionable crosstalk or noise in physical circuit operation, may not

be sufficiently balanced for successful phantom operation.

Standard repeating coils designed for phantom sets are mounted in pairs on a single base. Each coil includes four windings, as shown schematically in Figure 240, and the ends of each winding are brought out to terminal lugs numbered as indicated. Where the coils are used on 2-wire circuits equipped with telephone repeaters, one of the coils is used as a balancing coil, as discussed in a later article. The windings 4-3 and 8-7, which are connected to form the line side of the phantom repeating coil, are carefully wound so as to be as nearly identical electrically as possible. This balance is required, as we have already seen, to avoid crosstalk within the phantom group. The windings 2-1 and 6-5 are not so well balanced and are therefore always connected to form the drop side of the repeating coil.

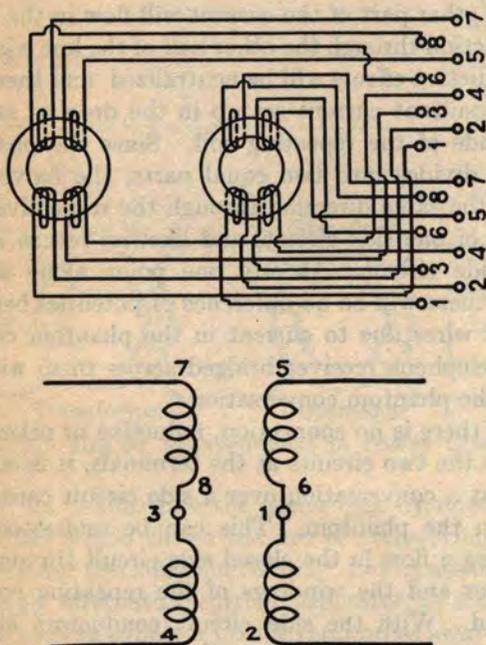


FIG. 240. WIRING DIAGRAM OF STANDARD REPEATING COIL

The code numbers of representative standard phantom repeating coils, having various impedance ratios as shown, are given in Table IX. The 93 and 75 type coils are identical except for different mounting arrangements. The same is true of the 62 and 85 type coils. In the manufacture of the 93 and 75 type coils, a core made of many turns of fine gaged silicon-steel wire is sawed so as to introduce a gap in the magnetic circuit. This gap is filled with a compressed powdered magnetic material, which while increasing slightly the core's reluctance, gives it a high degree of magnetic stability, preventing permanent magnetization under abnormal service conditions. The two windings for the line side are wound on the core together to give the

TABLE IX  
PHANTOM REPEATING COILS

IMPEDANCE RATIO LINE TO DROP 4-3 & 8-7 TO 2-1 & 6-5	SUITABLE FOR 20-CYCLE SIGNALLING		NOT SUITABLE FOR 20-CYCLE SIGNALLING	
	Relay Rack	Coil Rack	Relay Rack	Coil Rack
1:1	93-A	75-A	62-A	85-A
1:1.62	93-B	75-B	62-B	85-B
1.62:1	93-F	75-F	62-C	85-C
2.66:1	93-G	75-G	62-E	85-E
1.24:1	93-H			
2.28:1	93-J			
1:1.28			62-F	
1:2.34			62-G	

required high degree of balance, but the drop windings are wound individually. The iron core of the coil is wrapped with cotton tape to protect the windings, and after the windings are put in place, the coil itself is given a wrapping of cotton tape. It is then impregnated with a moisture proof compound, placed in its case, and melted resin is poured around it until it is firmly imbedded. The leads are then brought out to the terminal punchings.

The 85 and 62 type coils are made in the same ratios as the 93 and 75 type coils, and have approximately the same electrical characteristics. Their cores are made in the same way as described above, except that the gap in the magnetic circuit is not filled with compressed iron powder. This feature makes these types of coils somewhat more stable and they are therefore especially well adapted for use in circuits on which rapidly changing direct currents are superposed, such as those involved in high speed teletypewriter service. The same feature, however, tends to make these coils very inefficient at low frequencies and they cannot be used on circuits employing 20-cycle signaling.

### 116. Autotransformers

There is a type of transformer which may step-up or step-down the voltage, but does not employ a secondary circuit that is electrically separated from the primary circuit. It is called the "autotransformer", and the manner in which the primary and secondary windings are connected together electrically, as well as magnetically, is shown in Figure 241-A. As in any transformer, the primary and secondary windings are both wound on the same iron core so that any lines of magnetic induction that thread one winding also thread the other. The fact that the windings are directly connected does not prevent the voltages on the primary and secondary sides from being proportional to the number of turns in the windings, just as in the regular transformer. Thus, in Figure 241-A if there are 800 turns between *a* and *d*, 200 turns between *b* and *c*, and 110 volts are connected to *bc*, the primary,

the secondary voltage will be—

$$110 \times \frac{800}{200} = 440 \text{ volts}$$

This is a one-to-four step-up transformer, and if we should reverse the primary and secondary connections, we would have a four-to-one step-down transformer.

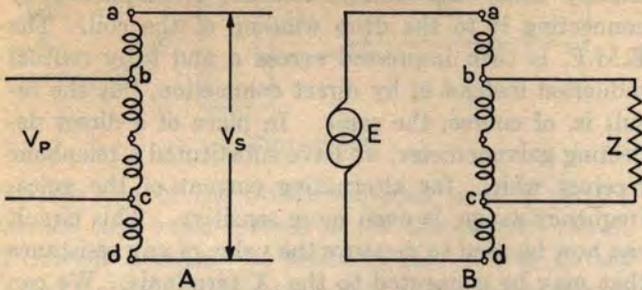


FIG. 241. THE AUTO-TRANSFORMER

Figure 241-B shows a step-down autotransformer with a voltage  $E$  connected to its primary and a load of impedance  $Z$  connected to its secondary. Since the winding  $bc$  is shunted by the load  $Z$ , it will be apparent that only part of the current in  $Z$  will flow through the secondary winding. The remainder will flow through the portion of the primary winding represented by  $ab$  and  $cd$ , and the generator. In other words, the primary current will flow through only a portion of the primary winding, and only a portion of the load current will flow through the secondary winding. In an ordinary transformer having entirely separate primary and secondary windings, on the other hand, all of the load current flows in the secondary winding, and all of the generator current flows in the primary winding. In a practical transformer the currents cause  $I^2R$  losses in the windings in which they flow in any case. But because of the fact noted above, the  $I^2R$  losses in the autotransformer are lower than in a transformer of the usual type, it being understood that the same size wire is used on each. This means that an autotransformer can be designed to have the same losses as a regular transformer, and still have less copper in the windings, thereby effecting a saving.

However, the direct electrical connection between windings is a disadvantage in power work since it introduces the hazardous possibility of obtaining the full primary voltage on the load side of the device if the secondary winding should become open.

In telephone work, it is sometimes desirable to use a transformer to change the effective impedance of a circuit without changing the circuit continuity for telegraph currents, D.C. signaling or D.C. testing. Figure 242 shows how this is accomplished through the use of an autotransformer; the condenser connected between the windings prevents any direct-current flow from one side of the circuit to the other.

### 117. The Hybrid Coil

In telephone repeater operation, as in duplex telegraphy, we must receive incoming energy and direct it into a receiving circuit (input) which is separate and distinct from the sending (output) circuit. This is essential inasmuch as the device used for amplifying voice-frequency currents can operate in one direction

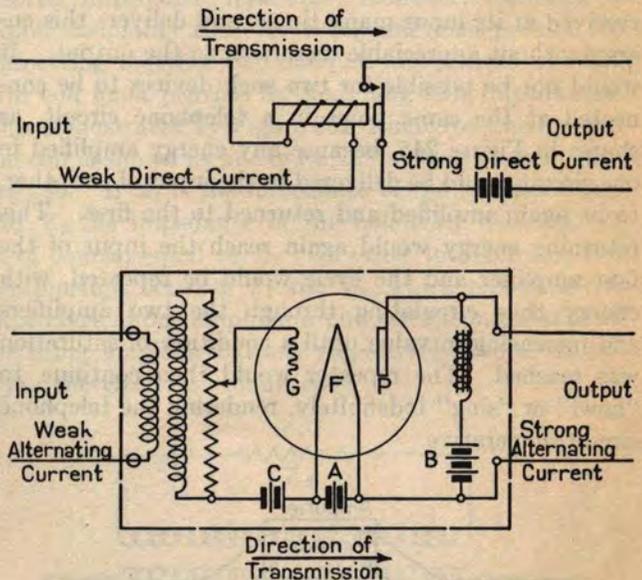


FIGURE 243

only. Its limitations in this respect are analogous to the telegraph relay, which repeats a direct-current signal from a circuit having a small amount of energy into one having a greater amount of energy (see Figure 243). The use of such one-way amplifiers, without some device for securing transmission in both directions, would be restricted to such a layout as is shown in Figure 244. This would require not only twice the circuit facilities for each long distance connection, but also special telephones at each terminal. We learned in Chapter XI how duplex telegraphy is accomplished over a single wire by application of the Wheatstone

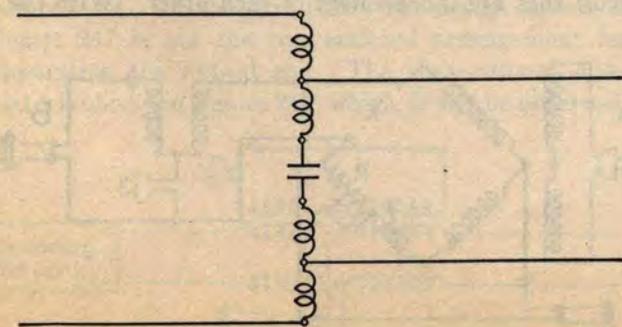


FIGURE 242

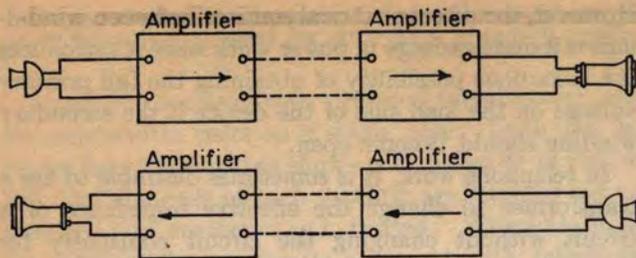


FIGURE 244

bridge principle and the use of an artificial line. The problem in the case of the telephone repeater is somewhat more complicated, but its solution is effected by employing the same principle of bridge balance, using an artificial line called a "balancing network".

In later chapters, we shall discuss the vacuum tube and its application in the telephone repeater. We may take up at this time the "hybrid coil", which makes two-way transmission over circuits equipped with one-way amplifying devices possible. For this discussion we may consider the amplifier circuit as a device which increases the energy of the voice current received at its input many times, and delivers this energy without appreciable distortion to the output. It would not be possible for two such devices to be connected at the same point in a telephone circuit, as shown in Figure 245, because any energy amplified in one circuit would be delivered to the input of the other, to be again amplified and returned to the first. This returning energy would again reach the input of the first amplifier and the cycle would be repeated, with energy thus circulating through the two amplifiers and increasing in value until a condition of saturation was reached. The repeater would then continue to "howl" or "sing" indefinitely, rendering the telephone circuit inoperative.

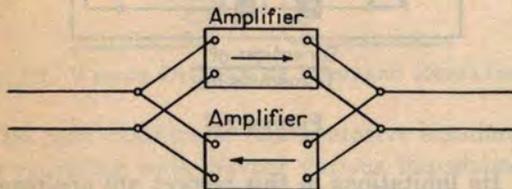


FIGURE 245

To eliminate the possibility of repeater singing, we must convert the ordinary telephone circuit into a receiving and a sending circuit which are independent of each other. That is, as in the case of the duplex set, the two circuits must be connected to the same line, yet any current flowing in one must not in any way affect the other. We can obtain this desired result by applying the principle of bridge balance, but the application is now to alternating currents. A

Wheatstone bridge with proper modifications, however, can be operated with alternating current as well as direct current. To illustrate, in Figure 246 we have a repeating coil connected as an alternating-current Wheatstone bridge. Here the source of voltage is an A.C. generator instead of a battery, and instead of connecting the voltage to the points *a* and *b* as is usually done, the same results are accomplished by connecting it to the drop winding of the coil. The E.M.F. is then impressed across *a* and *b* by mutual induction instead of by direct connection, but the result is, of course, the same. In place of a direct deflecting galvanometer, we have substituted a telephone receiver which, for alternating current of the voice-frequency range, is even more sensitive. This circuit can now be used to measure the value of any resistance that may be connected to the *X* terminals. We can

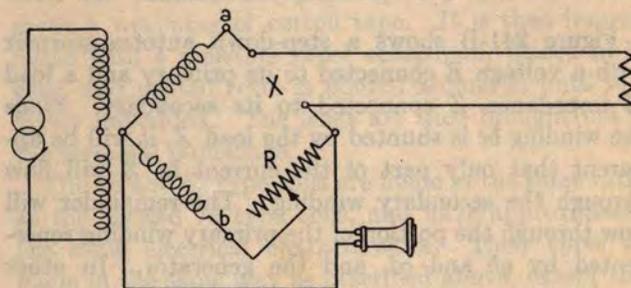


FIGURE 246

also use this circuit to measure any impedance that might be connected to the *X* terminals, provided the variable arm *R* has in series with it a variable reactance for balancing the reactive component of the unknown impedance.

Let us now assume that an alternating-current bridge circuit, such as that shown in Figure 246, but arranged to measure impedance as well as non-inductive resistance, has a transmitter substituted for its A.C. generator, and a telephone line terminating in a subset at the distant end, connected to the *X* terminals. Such an arrangement is illustrated by Figure 247. Here we have a device for terminating an ordinary telephone circuit so as to provide a receiving and a sending circuit that are independent of each other. With the

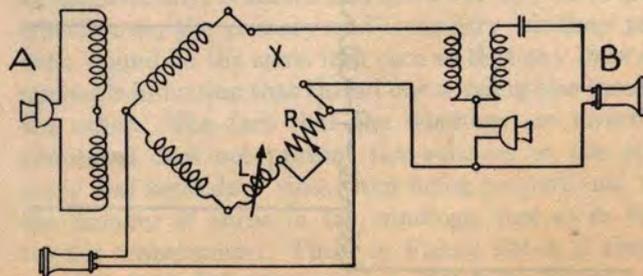


FIGURE 247

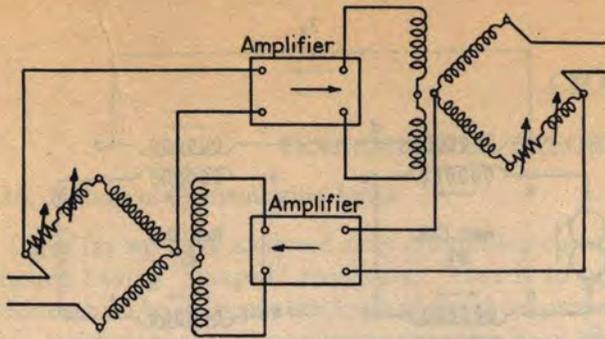


FIGURE 248

variable arm of the bridge adjusted to give perfect balance, any voice current in the transmitter circuit at Station A cannot be heard in the receiver circuit at that station, for the same reason that a galvanometer needle is stationary in any balanced bridge. We have double-tracked, so to speak, the ordinary 2-way telephone circuit.

If we now take two such circuits, and introduce two amplifiers as shown in Figure 248, we have a device that may be used as a telephone repeater at some intermediate point in a 2-wire telephone circuit. Here the energy coming from one amplifier cannot find its way into the input of the other and cause singing.

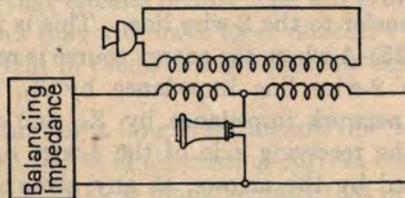


FIGURE 249

The coil that takes the place of the bridge mechanism in Figures 247 and 248, is known as a hybrid coil, sometimes called bridge transformer, three-winding transformer, repeater output transformer, etc. In the actual coil, there are a few additional details of design that do not permit the identity of the simple A.C. bridge circuit to be so readily recognized. These are not difficult to follow, however, after having been once pointed out. In the first place, the design shown in Figure 247 is not the conventional arrangement for illustrating the hybrid coil. The conventional schematic is shown in Figure 249, which, it will be observed

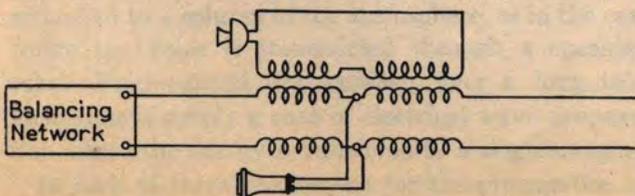


FIG. 250. THE HYBRID COIL

shows the same circuit connections as Figure 247 but is less similar to the standard convention for the Wheatstone bridge. In the actual hybrid coil, the line coils are divided and connected on both sides of the line as shown by Figure 250, in order that perfect symmetry in the wiring of the talking circuit may be maintained. Both sets of windings, of course, are inductively coupled to the third winding. Figure 251

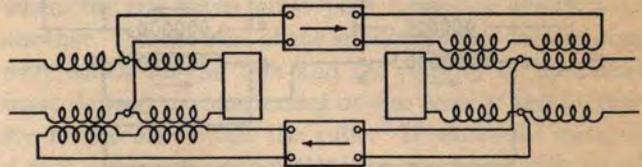


FIG. 251. TWO-WIRE TELEPHONE REPEATER CIRCUIT

shows the revised schematic of the amplifier connections to two hybrid coils in a 2-wire telephone repeater circuit.

In the hybrid coil, as in other transformers or repeating coils, the design must be such as to give the desired impedance relations. However, although a simple inequality ratio repeating coil must provide for connecting together two unequal impedances, the hybrid coil must provide for matching four impedances. This is illustrated by Figure 252, where for convenience the coil is shown as in Figure 249 instead of as in Figure 250. If  $Z_1$  is the impedance of the telephone line and  $Z_2$  the impedance of the balancing network,  $Z_1$  is, of course, equal to  $Z_2$ . In order to determine the relationships between  $Z_3$  and  $Z_4$ , which represent the impedance of one amplifier input and the impedance of the other amplifier output, respectively, we must analyze the electrical conditions.

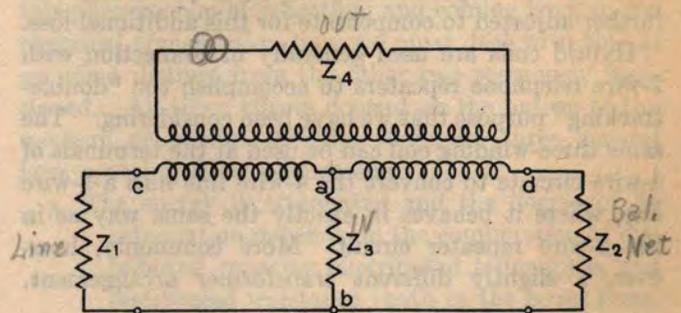


FIGURE 252

If we represent the source of voltage in the output circuit by a generator connected in series with  $Z_4$ , the energy supplied to the coil will obviously divide equally at the bridge, one-half going to each of the two equal impedances,  $Z_1$  and  $Z_2$ . None will get to  $Z_3$ . The part going to  $Z_2$ , which represents the impedance of the network circuit, accomplishes no useful purpose and is lost. For this reason alone, the amplifier must

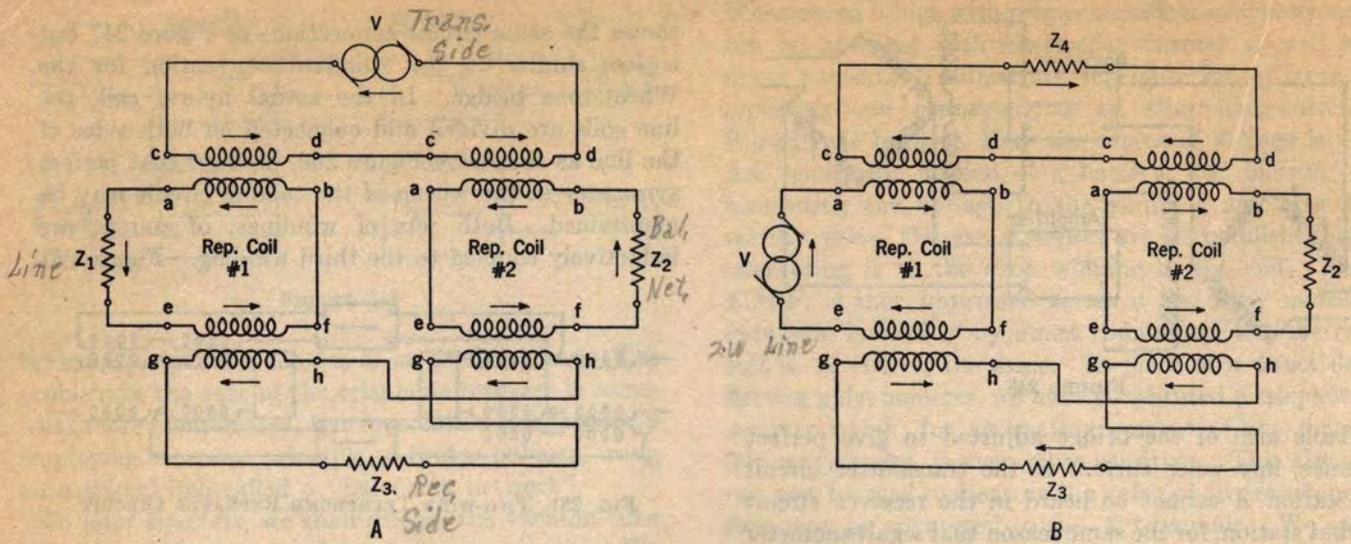


FIG. 253. 4-WIRE TERMINATING SET

be adjusted to supply twice the energy that is required for actual transmission. If, now, we simulate the conditions for inward transmission, connecting the generator in series with  $Z_1$ , the coil relations are such that half the energy goes to  $Z_3$  and half is dissipated in  $Z_4$ , but none reaches  $Z_2$ . The voltage induced between  $c$  and  $a$  is equal to the voltage induced between  $a$  and  $d$  because the windings have the same number of turns and are on the same magnetic core. The turn ratio of the coil is fixed at such a value that the voltage induced in the latter winding is just equal to the voltage drop across  $Z_3$ . Consequently, points  $b$  and  $d$  are at the same potential. There is therefore no current flow between these points, and  $Z_2$  consumes no energy. As before, however, half the incoming energy is lost in the impedance  $Z_4$ , so the amplifier must be further adjusted to compensate for this additional loss.

Hybrid coils are used generally in connection with 2-wire telephone repeaters to accomplish the "double-tracking" purpose that we have been considering. The same three-winding coil can be used at the terminals of 4-wire circuits to convert the 4-wire line into a 2-wire line, where it behaves in exactly the same way as in the 2-wire repeater circuit. More commonly, however, a slightly different transformer arrangement,

known as a "4-wire terminating set", is used for this purpose. This consists of two ordinary repeating coils connected with one winding reversed, as shown in Figure 253.

The principle involved here is the same as for the hybrid coil proper, as may be seen by analyzing the circuit. Thus, we may consider first the case of energy coming from the transmitting side of the 4-wire line for transfer to the 2-wire line. This is illustrated by Figure 253-A where the energy source is represented by  $V$ , the 2-wire line impedance by  $Z_1$ , the equal balancing network impedance by  $Z_2$ , and the impedance of the receiving side of the 4-wire line by  $Z_3$ . As indicated by the arrows, at any given instant  $V$  sets up equal voltages in  $Z_1$  and  $Z_2$ , but because the winding  $g-h$  of repeating coil 2 is reversed, the voltage set up in this winding is opposed by the equal voltage set up in winding  $g-h$  of repeating coil 1. As a result, no current is established in  $Z_3$ . Similarly, where the energy comes from the 2-wire line, as illustrated in Figure 253-B, equal voltages are set up in  $Z_3$  and  $Z_4$  and there is no current in the network,  $Z_2$ . This is because the direction of the voltage set up in winding  $e-f$  of repeating coil 2 is such as to oppose the equal voltage set up in winding  $a-b$ .

## CHAPTER XIX

### TRANSMISSION THEORY OF LONG TELEPHONE LINES

#### 118. Nature of Transmission Lines

Thus far we have analyzed only alternating-current circuits having "lumped" constants. That is to say, whenever we have encountered one of the three properties, resistance, capacity or inductance, we have considered it as pertaining to a specific piece of apparatus having a definite location in space. The only capacity we have known has been that which was a property of some device of definite size such as a condenser, and we have been able in every case to connect directly to the terminals of such a device. The same may be said of each resistance and inductance. This has simplified the make-up of the networks we have considered. In taking up the long transmission line, however, we shall find a different set of conditions. Though we shall not encounter any properties other than capacity, resistance and inductance, these will not be lumped. They will be more or less uniformly distributed along the entire length of the line, in fact they will be almost inseparably distributed. We can naturally expect, therefore, that circuits of this type will exhibit certain peculiarities that will make more difficult the analysis of the current in them, which represents energy transmission.

The nature of a long transmission line to which is connected a source of alternating-current energy, or an alternating E.M.F., is fundamentally that of a **medium** for wave propagation. It is another manifestation of the various forms of energy we have about us in all nature such as sound, heat, and light, being transmitted through some medium, though in this case we are dealing with electrical waves rather than sound, heat, or light waves. We speak of this form of transmission as "propagation". In all forms of propagation, the energy is in the form of moving waves and encounters opposition at every point in the medium. This tends to dissipate or cause the energy to die out, and we speak of this as the "attenuation" of the energy. A typical illustration is the case of sound energy being transmitted through the atmosphere. The attenuation is lower to some extent if the sound energy is restricted to a **column** of the atmosphere, as in the case where the voice is transmitted through a speaking tube. Voice-current transmission over a long telephone line is simply a case of electrical wave propagation where the energy is restricted to a single channel.

In each of these phenomena for the propagation of the various forms of energy, both the degree of atten-

uation and the speed at which the wave travels through the medium depend upon the nature of the medium. Furthermore, there are certain reactions that take place whenever the wave must pass from one medium to another. In the case of the speaking tube, the distance over which we can talk and the velocity of the sound wave depend to some extent on the density and humidity of the air within the tube. If we could imagine a case where one end of the tube was filled with air of a different density and degree of moisture saturation from that at the other end, we might hear an echo at the speaking end, due to a part of the energy being reflected back at the junction of the two transmitting mediums.

Perhaps a better illustration of the reflection phenomenon is the case of light, which has a definite velocity through the atmosphere but when it strikes a clear body of water such as a still lake, travels slower in the water than in the atmosphere. By our own observation, we know on the one hand, that this light may continue through the water until it illuminates pebbles on the bottom of the lake, while on the other hand, we find a mirrorlike reflection on the surface of the still water and know that some light is being reflected as it strikes the surface, in the same way that light is reflected when it strikes the surface of a mirror. It is only reflected in part, however, as we have evidence that some of the light has penetrated the more difficult medium. In all forms of wave motion we may have this phenomenon of reflection, and coming back to our electrical transmission line, we must deal with this as an effect distinct from the other two previously mentioned. All three effects depend on the nature of the medium or media. Briefly, there are three general laws covering these phenomena:

- a. The energy is attenuated and the degree of its attenuation depends on the combination of distributed capacity, distributed inductance, and distributed resistance (both in the series form, as that of the conductors and in the shunt form, as that of leakage).
- b. There is a definite speed at which the wave travels, which depends upon the electrical characteristics of the transmission line as established by the properties mentioned in *a* above.
- c. There is a reflection of energy whenever the wave passes the junction of one transmission line with another, if the two lines have different electrical characteristics.

To analyze alternating-current flow to the most accurate degree under such conditions, where we have wave propagation rather than simple flow through a localized circuit with lumped properties, and must take into consideration the circuit properties as they exist and the conditions brought about by the uniform distribution of these properties over great lengths, would naturally involve the higher branches of mathematics. For most practical purposes, however, and for the applications that we meet in everyday telephone work, we may closely simulate or approximate the electrical make-up of any transmission line by some form of circuit having lumped properties.

In order that we may obtain a clear idea of the processes involved in as simple a manner as possible, we may profitably first consider this general problem on a direct-current basis. In doing this we will need to remember that such a treatment is largely hypothetical, as both telephone and telegraph transmission are essentially alternating-current phenomena; but we will, nevertheless, be able to establish certain general principles more easily than by handling the problem as an alternating current one from the beginning. Then having established these principles, we may revert to our alternating-current transmission problem and make such modifications as are necessary in order that they may apply equally well to the alternating-current case.

TABLE X

THE COMPARATIVE PERCENTAGES OF POWER DELIVERED TO A RECEIVING DEVICE FOR VARIOUS RATIOS OF ITS RESISTANCE TO THE INTERNAL RESISTANCE OF THE SUPPLY SYSTEM, AND THE EFFICIENCY AT WHICH POWER IS SUPPLIED TO THE RECEIVING DEVICE FOR THE SAME RATIOS

VALUE OF $R_2$	% OF MAXIMUM $P_2$ $= 100 \times \frac{4R_2R_1}{(R_1 + R_2)^2}$	% EFFICIENCY $= \frac{100}{\frac{R_1}{R_2} + 1}$
2.0 $R_1$	88.9	66.7
1.1 $R_1$	99.8	52.4
1.0 $R_1$	100.	50.0
.9 $R_1$	99.7	47.4
.5 $R_1$	88.9	33.3
.2 $R_1$	55.6	16.7

### 119. The Transmission System

Any transmission system consists of three essential parts; a source of energy, a medium over which it is desired to transmit energy to a receiving device, and the receiving device itself, which usually converts the electrical energy into some form more useful. In a power transmission line, an electrical generator may be the source of energy; high voltage lines with transformers at either end may be the transmitting medium; a motor, lamp, or heater may be the receiving device

for converting electrical energy into some other useful form. In a long distance telephone connection, a transmitter may be considered as the source of energy; the line from the speaking party to the listening party with all of its associated conductors, coils, and connections, may be thought of as the transmission medium, and the telephone receiver at the distant end may be considered as the third part of the transmission system, or the device which converts tiny electrical currents into audible vibrations of air called sound waves. Regardless of the kind of system, we must have these factors.

### 120. Transfer of Power

If a transmission system is to accomplish its purpose, it must be so designed that the energy transmitted from the source to the receiving device is sufficient to successfully operate the receiving device. As a secondary consideration it may be designed for power efficiency that is, regardless of the magnitude of the power delivered to the receiving device, the power lost in transmitting the energy from the source must be kept at a minimum. Although this is important in any transmission system, its special importance is in power transmission. In telephone work we probably think more of the primary purpose, that is, the system's effectiveness in transferring the maximum quantities of power to the receiving device, regardless of what percentage may be lost.

To illustrate the principles of power transfer and power efficiency, let us consider a small direct-current power distribution system. Such a system is usually a complicated network, consisting of a combination of many series and parallel resistances. When connecting a lamp to the lighting mains, we are concerned primarily in the transfer of power to the lamp. The lamp then, is the receiving device; the wiring from the lamp to the mains is the transmitting medium, and the mains are the source of energy. Looked at in this manner, the source of energy is no longer a simple device such as a battery, but is itself an energized network of complex make-up. Moreover, the current and voltage distribution in this energized network are influenced by the presence or absence of the lamp; current and voltage values elsewhere in the system will change as the lamp is connected to or disconnected from the mains. We know that our receiving device has a constant resistance, and for a constant voltage, draws a definite current. We further know that the power that is expended in the device is equal to its resistance multiplied by the current squared. This we may call the useful expended power. But if the current coming from the source of electromotive force, must traverse other resistances or other parts of the complicated network,

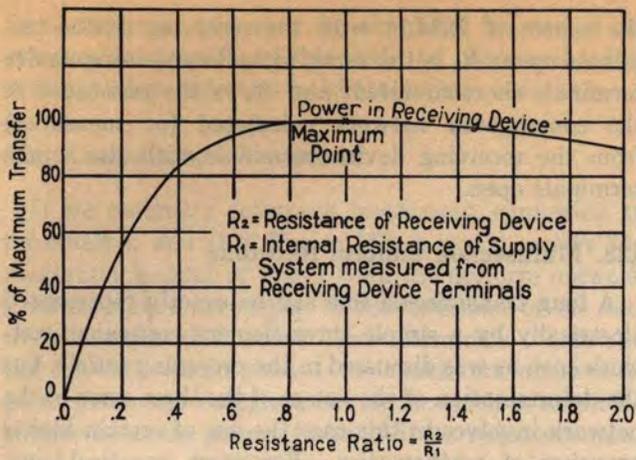


FIG. 254. POWER TRANSFER AS A FUNCTION OF RESISTANCE RATIO

as is the usual case, part of the power which is actually delivered by the source because of the connection of the particular receiving device, will be lost in the distribution system. The ratio obtained by dividing the power received by the device by the power expended by the source on account of its connection, is called the **power efficiency**. This will increase with increase in resistance of the receiving device. Other things being equal, therefore, we have the most efficient operation when the receiving circuit is one of very high resistance.

On the other hand, we may be interested in receiving all of the power possible, regardless of whether the operation under such circumstances is efficient or not. In the case of a telephone receiver at the end of a long transmission line, we are primarily interested in the receiver taking from the electrical system the maximum amount of power. **The condition for maximum transfer of power is obtained when the resistance of the receiving circuit is equal to the resistance of the network to which it is connected, as measured across the receiving terminals.** The simplest application of this is secured by connecting to a battery a resistance equal in magnitude to the internal resistance of the battery. In this case the battery will transfer to the external

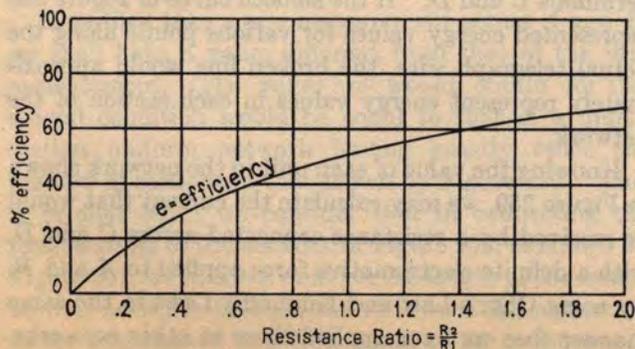


FIG. 255. EFFICIENCY AS A FUNCTION OF RESISTANCE RATIO

circuit the maximum amount of power, but in doing so will operate at an efficiency of only 50 per cent.

Figure 254 shows a curve which represents the power of the external circuit for various ratios of the resistance of the external circuit to that of the internal circuit. Figure 255 shows the efficiency for the same conditions. Table X gives the values from which these curves are plotted.

### 121. Pollard's Theorem

For the purpose of simplifying electrical calculations, we can consider any electrical system as one network supplying energy to another. One or the other of these networks may then be replaced by an equivalent circuit of maximum simplicity. **For every energized network there is an equivalent simple electrical circuit which consists of an E.M.F. and a resistance in series.**

This means that regardless of how complicated an electrical circuit may be, its effect in supplying current to any other circuit connected to it at two designated terminals, is equivalent to some source of electromotive force in series with a resistance. In other words, it is equivalent to a source of electromotive force, such as a battery, having an internal resistance of a definite value. This principle is called **Pollard's Theorem** and Figure 256 illustrates its use.  $E$  is a source of electromotive force connected to a complicated network;  $A$  and  $B$  are terminals to a particular branch of the complicated network. If it is desired to connect some receiving device to these terminals, the effect of this electrical system on the receiving device will be the same as that of the electrical system shown by Figure 257 where  $E'$  is the electromotive force measured across the terminals  $A$  and  $B$  of Figure 256, and  $R'$  is the resistance measured or calculated from the same terminals with the electromotive force  $E$  short-circuited. Pollard's Theorem may be briefly stated as follows:

**The current supplied to an electrical device connected to two terminals of any electrical system is equal to the potential measured across these terminals before the device is connected, divided by the resistance measured or calculated across these terminals with the source of E.M.F. short-circuited, plus the resistance of the receiving device.**

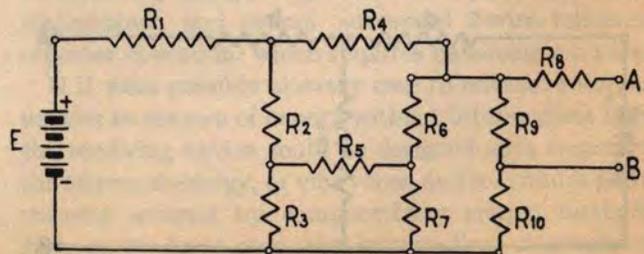


FIGURE 256

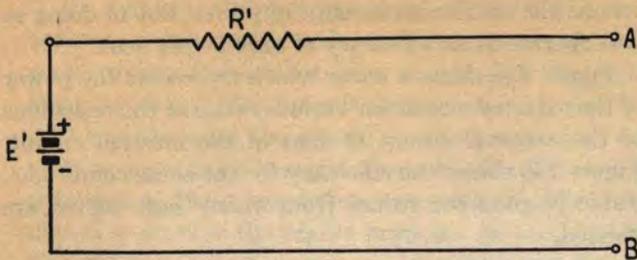


FIG. 257. APPLICATION OF POLLARD'S THEOREM TO THE NETWORK OF FIG. 256

### 122. Equivalent Networks

Pollard's Theorem gives us a method of substituting an equivalent circuit for any complicated electrical system, but in so doing we are required to replace the source of electromotive force with one having another value. It is often desired to determine the simplest equivalent network for a complicated electrical system which will supply to some receiving device the same current as the electrical system and will take from the same source of electromotive force the same current as the electrical system. A network consisting of three resistances of proper value arranged in the form of a T as shown by Figure 258 can always be substituted for any network, regardless of how complicated, and fulfill these conditions. For example, the circuit in Figure 258 may be substituted for that shown by Figure 256, and the current supplied to this system by the electromotive force  $E$  will remain unchanged, and the current received by a device connected to the terminals  $A$  and  $B$  will be the same. As we shall see in a later chapter, this same result can also be effected by means of a simple network having three arms arranged in the form of a  $\pi$ .

In determining values for the three resistances in an equivalent T-network such as is shown by Figure 258, the following equations may be used:

$$\text{Resistance of } a = R_1 - c \quad (72)$$

$$b = R_3 - c \quad (73)$$

$$c = \sqrt{(R_1 - R_2)R_3} \quad (74)$$

where  $R_1$  is the calculated (or measured) resistance of the complicated network at the terminals connected to

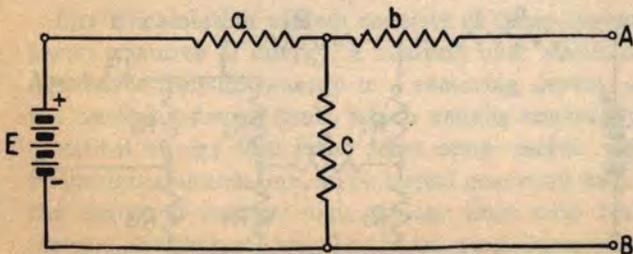


FIG. 258. T-NETWORK

the source of E.M.F. with the receiving device terminals open;  $R_2$  is the same with the receiving device terminals short-circuited; and  $R_3$  is the resistance of the complicated network calculated (or measured) from the receiving device terminals with the source terminals open.

### 123. Multisection Uniform Networks

A long transmission line can be exactly represented electrically by a simple three-element equivalent network such as was discussed in the preceding article, but the determination of the values of the three arms of the network involves in this case the use of certain higher branches of mathematics. For most practical purposes, we may deal with the transmission line by considering it as consisting of a number of separate sections. Treating at this time the direct-current case, we shall assume an approximately equivalent network for a transmission system such as a grounded telegraph wire 50 miles in length, and having a uniform leakage to ground throughout. We can imagine such a circuit as ten uniform sections, and for our purpose, may consider the leak to ground in each five-mile section as concentrated at the middle point. With these assumptions, our circuit may be represented by the network shown in Figure 259. Such a network is

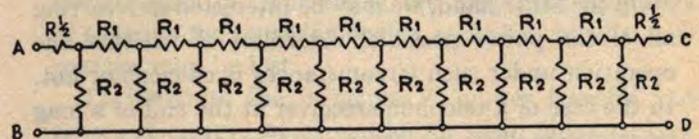


FIG. 259. MULTISECTION UNIFORM NETWORK

called a **multisection uniform network** because it consists of a number of identical units joined together. While a network thus constructed is not identical to the actual telegraph wire, we can construct one as nearly identical as we may desire by making our sections shorter in length. In this particular case, a 10-section uniform network would give a very small error in calculations for a receiving instrument connected across terminals  $C$  and  $D$ . If the smooth curve of Figure 260 represented energy values for various points along the actual telegraph wire, the broken line would approximately represent energy values in each section of the network.

Knowing the value of each unit in the network shown in Figure 259, we may calculate the current that would be received by a resistance connected across  $C$  and  $D$ , with a definite electromotive force applied to  $A$  and  $B$ , by using Ohm's Law and Kirchoff's Laws in the same manner that we have applied them to other networks. This procedure is rather laborious for long transmission

lines, however, and may be simplified by use of an **attenuation** formula. This will be discussed after defining what is meant by characteristic resistance.

## 124. Characteristic Resistance

If we assume a telegraph instrument connected to terminals *C* and *D* of Figure 259, it would receive the maximum amount of power from the uniform network if its resistance were equal to the resistance of the network as measured across these terminals. Likewise the network would receive the maximum amount of power from any energized circuit to which it might be connected at the points *A* and *B*, if its resistance measured across *A* and *B* were equal to the resistance of the energized circuit. This we learned from the principle of maximum power transfer. But before connecting an energized circuit for sending to *A* and *B* or a circuit for receiving to *C* and *D*, let us measure the resistance of the network at *A* and *B*, and then construct a simple resistance  $R_0$  of this measured value, and connect it to *C* and *D*.

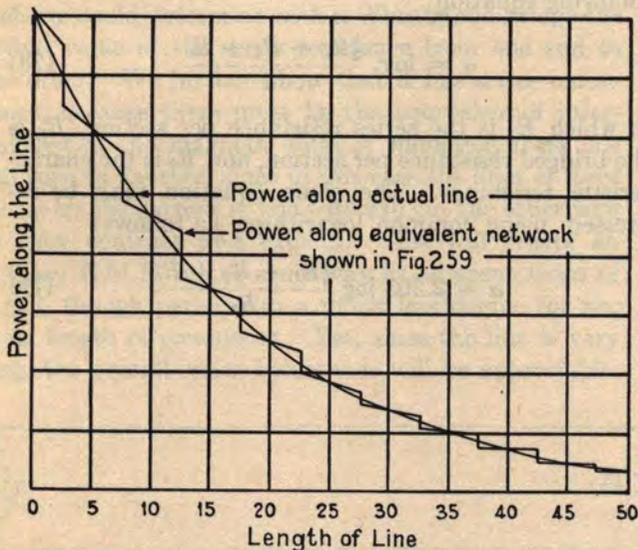


FIG. 260. COMPARISON OF POWER IN MULTISECTION UNIFORM NETWORK AND LINE WHICH IT SIMULATES

If we then take a new measurement across *A* and *B*, we shall have a value different from that of the first measurement. The value we would obtain for this second condition would be equal to that of a multisection uniform network having exactly twice the number of sections of that shown by Figure 259. This is evident when we consider that in connecting the resistance  $R_0$  to the network of Figure 259, as shown by Figure 261, we in effect doubled the length of the multisection network, because the resistance  $R_0$ , connected to the terminals *C* and *D*, is equal to the resistance of the network measured from *A* and *B* when the ter-

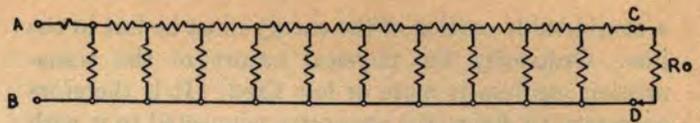


FIGURE 261

minals *C* and *D* are open. Therefore, Figure 261 is equivalent in all respects to a 20-section network open at the distant end, instead of the 10-section network shown by Figure 259. If the new measured resistance value is now substituted for the resistance  $R_0$  in Figure 261, we shall have a network equivalent to a 30-section uniform network. If we again take measurements and replace  $R_0$  with the still newer value and continue this practice indefinitely, each time in effect increasing the length of the network by ten sections, we will eventually have the equivalent of a line so long that anything connected to its distant end will have no effect upon the current which the sending element delivers to the line at the terminals *A* and *B*. Either short-circuiting or opening the distant end will not affect the equivalent resistance of the line.

This equivalent resistance, or the resistance of a network having an infinite number of sections, is called the **characteristic resistance**. Its value can be calculated from the relationship—

$$R_0 = \sqrt{\frac{1}{4}R_1^2 + R_1R_2} \quad (75)$$

where  $R_1$  and  $R_2$  are the elements of a network as shown in Figure 259, and  $R_0$  is its characteristic resistance. If the resistance of the receiving device is made equal to  $R_0$  and if the sending circuit supplying energy to the line is so designed as to have the same resistance as  $R_0$ , we shall have the conditions of maximum energy transferred both from the sending circuit into the line at *A* and *B*, and from the line into the receiving telegraph instrument at *C* and *D*. While in practice this condition may not be generally applied to telegraph operation, it does apply to long distance telephone circuit operation. Characteristic resistance bears the same relation to a direct-current transmission line as characteristic impedance bears to an alternating-current transmission line. In both cases, the principle is the same. It is paramount in the operation of long distance telephone circuits from two viewpoints—first, an efficiently designed system for simple voice-current transmission and second, successful 2-wire telephone repeater operation, which requires balancing networks.

If it were possible in every case to connect receiving devices to sources of energy without intermediate lines, the receiving device could be designed with respect to the source of energy, or vice versa, and maximum power transfer secured by comparatively simple methods. But, as we have seen, the intermediate transmission line complicates the problem; especially when at best

a considerable portion of the energy must be lost in the line. Ordinarily the physical nature of the transmission medium is more or less fixed. It is therefore necessary to design the apparatus connected to it with respect to the characteristic resistance (or impedance) of the line rather than to design one unit with respect to the other.

### 125. Attenuation

This subject, too, has little importance when applied to direct-current circuits such as the telegraph circuit previously discussed, but it likewise may be treated more simply from the direct-current aspect. We shall examine at this time, therefore, what is meant by attenuation, and the use of attenuation formulas in calculating current or voltage values at points along, or at the distant end, of a transmission line. If in Figure 261 the multisection uniform network has an infinite number of sections or is terminated in its characteristic resistance,  $R_0$ , the ratio of the current leaving any one section to that entering the section will be the same, regardless of what section is considered. That is,

$$\frac{I_2}{I_1} = \frac{I_3}{I_2} = \frac{I_4}{I_3} = \text{a constant} \quad (76)$$

To illustrate this, let us assume that the current entering the network at  $A$  and  $B$  is decreased at the end of the first section to a given fractional value, for example  $\frac{1}{2}$ ; the remaining current will be likewise decreased one-half to a value of one-quarter of the original at the end of the second section. In the same way, the current will be reduced to one-eighth of the original value at

the end of the next section, to one-sixteenth at the end of the following section, and so on indefinitely. This "dying out" or attenuation is due to a part of the current in each section returning through the shunting resistance instead of flowing toward the receiving end, and thereby becoming lost in so far as transmission from one end of the network to the other is concerned. If, for example, we desire to calculate the current value at the distant end of a 10-section uniform network we must multiply the ratio of the current entering each section to that leaving each section by itself 10 times, or take the 10th power of the fraction  $I_2/I_1$ .

Such calculations are usually made by the use of logarithms. This permits an equation to be written giving the ratio of the current at the receiving end,  $I_n$ , to the current at the sending end,  $I_1$  as follows:

$$\frac{I_n}{I_1} = \frac{1}{e^{n\alpha}} \quad (77)$$

where  $n$  is the number of sections,  $e$  is the base of the Naperian logarithm system, and  $\alpha$  is the **attenuation constant**. The value of  $\alpha$  can be calculated from the following equation:

$$\alpha = \log_e \frac{\frac{1}{2}R_1 + R_2 + R_0}{R_2} \quad (78)$$

in which  $R_1$  is the series resistance per section,  $R_2$  is the bridged resistance per section, and  $R_0$  is the characteristic resistance. The same equation may be expressed, using common logarithms, as follows:

$$\alpha = 2.303 \log \frac{\frac{1}{2}R_1 + R_2 + R_0}{R_2} \quad (79)$$

## CHAPTER XX

### TRANSMISSION THEORY OF LONG TELEPHONE LINES—(Continued)

#### 126. The Transmission Line as a Multisection Network

Now having made use of a direct-current analysis of the general problem of transmission over long distances to establish certain definitions and methods of attack, we may turn our attention to the less artificial but somewhat more complex problem of alternating-current transmission. Let us assume that Figure 262 represents a very long telephone line with alternating-current energy (such as that coming from a telephone transmitter) applied at one end, and some receiving device of impedance  $Z_R$  connected at the distant end. We know that such a line has series resistance. By short-circuiting the receiving device at the distant end, we could determine with a Wheatstone bridge the actual value of the series resistance from one end to the other. We further know that it has series inductance, because there must be the equivalent of interlinkages of the magnetic lines of induction from one coil turn to another, since in this case the lines of force set up by the current in one wire will cut the other wire as they contract and expand. This will create an induced E.M.F. in the same way as adjacent loops of a coil, though perhaps to a much less degree for the same length of conductor. Yet, since the line is very long, the overall series inductance will be appreciable.

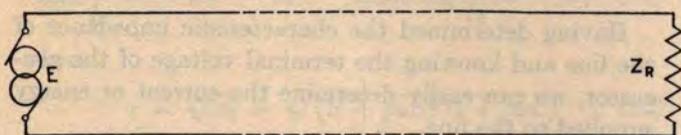


FIGURE 262

We further know that the line has some leakage which in any practical case will depend upon atmospheric conditions, but the insulation will never be so perfect that some leakage cannot be detected with a sufficiently sensitive instrument. There is one other property of the line. If it is open at the distant end, it will be found to act very much like a condenser. When a battery in series with a sensitive meter is connected to it, there will be a throw of the needle showing that the line temporarily is taking current to charge the two wires as though they were plates of a condenser.

Now let us assume that we know the resistance, inductance, leakage, and capacity of each mile of the

circuit, and also its total length. If we evaluate the constants of the circuit in its entirety and attempt to use these values directly to build a simple network that will simulate the line, we will find the task impossible. Even a T-network made up of these "nominal" values will fail to simulate the line if the latter be of any great length; and the greater the length, the greater will be the electrical dissimilarity between line and network. We could, of course, construct an equivalent T-network, using constants determined by measurements as explained in the preceding chapter, which would exactly simulate the line, but we should find this T quite unlike the nominal T. The relationship between the two networks would not be a simple one and would necessitate the use of "hyperbolic trigonometry" for its determination. However, by taking shorter sections of line to simulate, we find a closer agreement

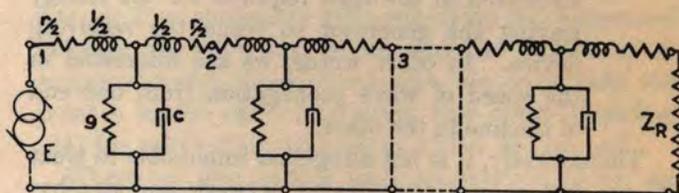


FIGURE 263

between the line and the nominal T, so that by considering the line as made up of a large number of extremely short sections, and constructing a multisection uniform network as illustrated in Figure 263, we can approximately simulate the line.

The degree of accuracy to which we approximate the line is going to depend on how far we go in breaking up the quantities into smaller parts. To begin with, we shall take an extreme case. Let us assume, for instance, that we have a circuit 1,000 miles long and are going to construct a network section for each foot, giving more than 5,000,000 sections for the network. Certainly we could not question the accuracy with which such a multisection uniform network would approximate the actual line. Assuming, therefore, that we have succeeded in so breaking up our distributed properties into tiny lumped properties which can be connected into a form of network, let us now accept this network, as illustrated by Figure 263, as equivalent for all practical purposes to the actual transmission line illustrated by Figure 262.

Now our interest in this network lies in—

- a. The current that will leave the generator at the sending end and flow into the network. This will be determined solely by the impedance the network presents at the “voltaged” end, when connected to the generator. If  $V_0$  is the terminal voltage of the sending device, and  $I_0$  the entering current, then  $I_0 = V_0/Z_0$  where  $Z_0$  is called the “sending end impedance” of the line (when the line is infinitely long it is the “characteristic impedance” of the line). Since our line is 1,000 miles long and will for all practical purposes draw the same sending current from the generator as an infinite line, we can consider  $Z_0$  in this case as the characteristic impedance.
- b. We are next interested in what part of the energy leaving the generator will eventually reach the receiving device at the distant end. Or, since energy depends on both voltage and current, we are interested in what part of the generator’s voltage will be impressed across the terminals of  $Z_R$  or what part of the generator’s current will flow through  $Z_R$ .
- c. For many transmission considerations we are also interested in the time required for the energy leaving the generator to reach the receiving device. In other words, we are interested in the speed of wave propagation from one end of the line to the other.

Theoretically, it is not altogether impossible to treat Figure 263 as any complicated network and step-by-step to calculate the impedance  $Z_0$ , as long as the number of sections is finite. In this case we are dealing with 5,000,000 sections, and it would be possible to calculate the current in each branch of the network or even through the distant receiving device, but certainly such extended computations would be impracticable and almost endless. The calculations for uniform multi-section networks are never made in this laborious manner. By a certain mathematical analysis, we derive short-cuts, based upon the following:

Knowing the make-up of the network sections, we might describe each as a series impedance  $z$  representing the series resistance and inductance of one foot of line, and a bridged impedance  $z_s$ , representing the bridged leakage and capacity of one foot; or instead of using  $z_s$ , we may for convenience use its reciprocal, which is called admittance and designated by the symbol  $y$ . We may express  $z$  and  $y$  in terms of the resistance, inductance, conductance (or leakage) and capacity for one foot of line. Let us represent these latter four quantities by  $R$ ,  $L$ ,  $G$  and  $C$  respectively. Here it should be noted that  $C$  is in farads and not micro-

farads. Now, the series impedance contains  $R$  and  $L$  and is given by the equation—

$$z = R + j\omega L \quad (80)$$

where  $z$ ,  $R$  and  $L$  are as defined above,  $\omega$  is equal to  $2\pi f$ , and  $j$  is an operator indicating  $90^\circ$  rotation as discussed in Article 109.

In the same manner, since  $G$  and  $C$  are bridged properties, we may write

$$y = G + j\omega C \quad (81)$$

An inspection of the make-up of each section would lead us to expect the characteristic impedance,  $Z_0$ , to become greater as  $z$  becomes greater, for the series impedance is tending to decrease the current which the generator attempts to establish. We should also expect an increase in the impedance of the shunt across each tiny section, the admittance of which we have designated as  $y$ , to permit less current to be shunted at each section and returned to the generator, thereby in its overall effect decreasing the amount of current that the generator would feed into the network. In other words, we should expect the quantity  $Z_0$  to become greater as  $z_s$  becomes greater, or as  $y$ , which is the reciprocal of  $z_s$ , becomes smaller.

The value of  $Z_0$  will tell us something of the nature of our transmitting medium, and since it is called characteristic impedance, it corresponds to the term “characteristic resistance” of the D.C. line, as discussed in Article 124. It can be shown that the value of  $Z_0$  for an infinite line may be determined from a relatively simple equation, as follows:

$$Z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (82)$$

Having determined the characteristic impedance of the line and knowing the terminal voltage of the generator, we can easily determine the current or energy supplied to the line.

Our next interest, as stated by *b* in the foregoing, is the part of this current or energy which will eventually reach the receiving device. Clearly it would be endless to proceed with ordinary network calculations, but again the calculations are simplified if we know **the degree to which each section of the network causes the current wave propagated along the line to die out.** Knowing this, we may say that the same attenuation when the line is treated as infinite applies to the voltage, because **the impedance of an infinite line is always the same when looking away from the sending end regardless of what junction of sections may be considered.** To illustrate, if we should open the multi-section network of Figure 263 and measure the impedance looking away from the generator at point 3,

we would get the same result as if we measured the impedance connected to the generator. We would get the value  $Z_0$ , which is the characteristic impedance of the line. Since  $Z_0$  always remains the same and must always be equal to  $V/I$  at any point along the network,  $V$  and  $I$  must be attenuated in the same ratio. If  $I$  becomes one-half of its value at some point along the line, then  $V$  must become one-half of its value, etc.

Now as we noted in Article 125, inasmuch as all sections are identical in their make-up, it can be seen that the loss or attenuation in each section will be the same, so that if the ratio of entering current to leaving current for the first section is nine-tenths, the ratio of currents for the second section and any succeeding section will be nine-tenths. Thus if we know the ratio of the current at point 2 to the current supplied the section by the generator, which we may represent by  $I_1/I_0$ , and wish to find the current at some point along the line, we can multiply this ratio by the succeeding ratios for each section as follows:

$$\frac{I_n}{I_0} = \frac{I_1}{I_0} \times \frac{I_2}{I_1} \times \frac{I_3}{I_2} \times \dots \times \frac{I_n}{I_{n-1}}$$

or since

$$\frac{I_1}{I_0} = \frac{I_2}{I_1} = \frac{I_3}{I_2} \text{ etc.}$$

$$\frac{I_n}{I_0} = \left[ \frac{I_n}{I_{n-1}} \right]^n \quad (83)$$

This is sometimes written  $k^n$  where  $k$  is the value of any one of these ratios, but for convenience in computation, the ratio is usually expressed by logarithms—

$$\log_e \frac{I_n}{I_0} = -n\gamma \quad \text{or} \quad 2.303 \log \frac{I_n}{I_0} = -n\gamma \quad (84)$$

where

$$\gamma = \sqrt{zy} = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (85)$$

and  $n$  denotes the number of sections traversed by  $I_n$ . Here we have a mathematical short-cut for our network calculations, which expressed in words, is as follows: **If we wish to know the relation between the current at any point along a transmission line and that delivered by the generator at the sending end, we can multiply the propagation constant of one section,  $\gamma$ , by the number of sections traversed, and the product taken negatively is 2.303 times the logarithm of the current ratio.**

But the quantity  $\gamma$  is more than a constant that gives the mere dying out effect of the current. The ratio of current  $I_n$  to  $I_0$  is a relation of both effective values and phase difference, as both  $I_n$  and  $I_0$  are vectors and are not necessarily in phase. This must be taken care of by treating  $\gamma$  as we treat all vectors; and  $\gamma$  is a vector

quantity because it is equal to  $\sqrt{zy}$  and both  $z$  and  $y$  are vectors. We must, therefore, separate the constant  $\gamma$  into two components, one of which applies to attenuation alone, and the other of which has to do with speed of propagation. We may write then, that—

$$\gamma = \alpha + j\beta \quad (86)$$

where  $\alpha$  is the symbol for the **attenuation constant** and  $\beta$  is the symbol for the **wave length constant**. Although it is practically always easier to evaluate  $\alpha$  and  $\beta$  by making use of Equations (85) and (86), it is possible to write equations giving their values directly in terms of the primary constants —  $R$ ,  $L$ ,  $G$  and  $C$ . These equations are as follows:

$$\alpha = \sqrt{\frac{1}{2} \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} + \frac{1}{2}(GR - \omega^2 LC)} \quad (87)$$

$$\beta = \sqrt{\frac{1}{2} \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} - \frac{1}{2}(GR - \omega^2 LC)} \quad (88)$$

In the foregoing discussions of Figure 263, we have in each case designated some point along the line such as point  $n$ , at which we wish to determine the current. This applies to an infinite line. If, however,  $Z_R$  is equal to  $Z_0$ , or in other words, if the line at the distant end is terminated in a receiving device having an impedance equal to the characteristic impedance of the line, or if an inequality ratio repeating coil is inserted between the receiving device and the line so as to properly match these impedances, we could take the point  $n$  as the distant terminal and apply Equation (84) for calculating the current at the distant end.

Where we are dealing with attenuation alone, we may express (84) as follows:

$$-n\alpha = 2.303 \log \frac{I_n}{I_0} \quad (89)$$

where  $I_n/I_0$  is the ratio of current magnitudes only. In other words, the ratio is now an arithmetic comparison between current delivered and current sent, ignoring the fact that there may be some phase difference between the two currents. To convert this to power, we can use the expressions—

$$P_0 = E_0 I_0 \cos \theta$$

and

$$P_n = E_n I_n \cos \theta$$

Now, for the infinite line—

$$\frac{E_0}{I_0} = Z_0 \quad \text{and} \quad \frac{E_n}{I_n} = Z_0$$

whence

$$\frac{E_0}{I_0} = \frac{E_n}{I_n} \quad \text{or} \quad \frac{E_n}{E_0} = \frac{I_n}{I_0}$$

so that

$$\frac{P_n}{P_0} = \frac{E_n I_n \cos \theta}{E_0 I_0 \cos \theta} = \frac{E_n I_n}{E_0 I_0} = \left[ \frac{I_n}{I_0} \right]^2 \quad (90)$$

Now Equation (89) can be squared to give—

$$2.303 \log \left[ \frac{I_n}{I_0} \right]^2 = -2n\alpha \quad (91)$$

and combining this with (90), we have—

$$2.303 \log \frac{P_n}{P_0} = -2n\alpha \quad (92)$$

also since

$$I_n = I_0 e^{-n\alpha} \quad \text{and} \quad E_n = E_0 e^{-n\alpha}$$

therefore

$$P_n = E_n I_n \cos \theta = E_0 I_0 e^{-2n\alpha} \cos \theta \quad (93)$$

The power, therefore, is seen to die out or attenuate in a ratio which is the square of the current ratio.

In the foregoing we find for the most part a mathematical significance of  $\alpha$  and  $\beta$ . Let us now analyze the physical circuit to determine what actually happens as the current is sent from point to point. In order to simplify the analysis, we shall start with an actual cycle of E.M.F. impressed on the sending end of a multisection network, and consider separately the effects of inductance and capacity on the propagation of this wave.

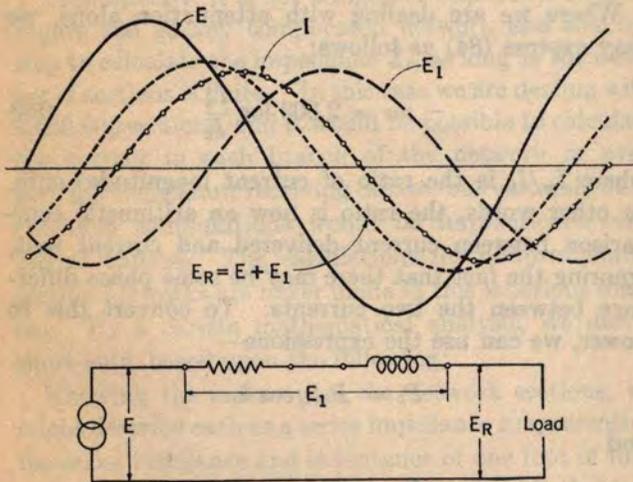


FIG. 264. VOLTAGES AND CURRENT IN AN INDUCTIVE CIRCUIT

From our previous study, we know that inductance acts to cause the current to lag behind the impressed voltage, so that in a circuit made up of resistance and inductance we would expect a lagging current. Figure 264 shows the time relationship between voltage and current in such a circuit, where  $E$  is the voltage curve, and  $I$  the current curve. This current sets up a back

or induced E.M.F.  $E_1$ , which is the sum of the  $IR$  drop across the resistance and the  $IX$  drop across the inductance. It combines with the original voltage  $E$  to give the resultant voltage  $E_R$  on the load side of the inductance. The curve  $E_R$  is obtained by adding  $E$  and  $E_1$  and it will be observed that the resulting curve lags  $E$ , the original voltage. A circuit containing resistance and capacity, on the other hand, produces a leading current as shown by Figure 265, and this current produces an  $IR$  drop which is opposite in phase with the current. Now if we combine the  $IR$  drop and the voltage, we obtain the resultant voltage  $E_R$ , which exists across the condenser and the load. This voltage likewise lags  $E$ , the original voltage.

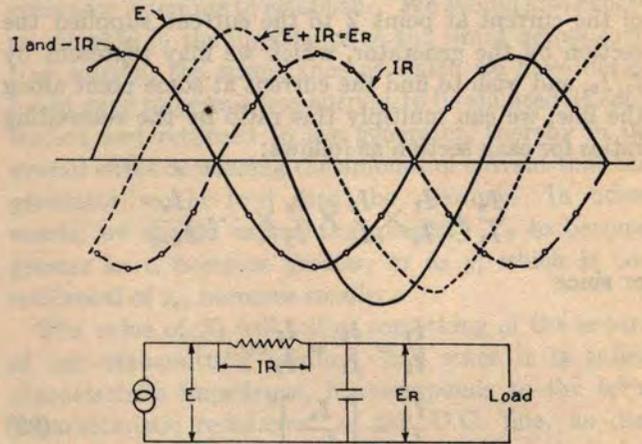


FIG. 265. VOLTAGES AND CURRENT IN A CAPACITIVE CIRCUIT

In both cases we have obtained a resultant voltage which lags behind the impressed voltage. Bridged capacity assists series inductance in the phase retarding effect. Due to the presence of reactance, therefore, the voltage has been "held back", so that the maximum voltages act later than they would if the reactance were removed. In other words, **the voltage wave has been slowed down**. Here, then, we have an explanation of the significance of the wave-length constant; it is merely an index figure to show how much the wave is retarded. Let us now apply our knowledge to the further study of the transmission line which we have represented by a series of T-sections. Each section, due to resistance and leakage, absorbs energy and therefore reduces the voltage which can act on the next section. Further, the voltage available at the next section lags behind the voltage impressed on the section, so that as we move away from the generator, the acting voltages are lagging farther and farther behind the generator voltage. Here we have a connecting link between geographical **distance** travelled along the line and **time**.

To bring this out clearly, let us assume that we take

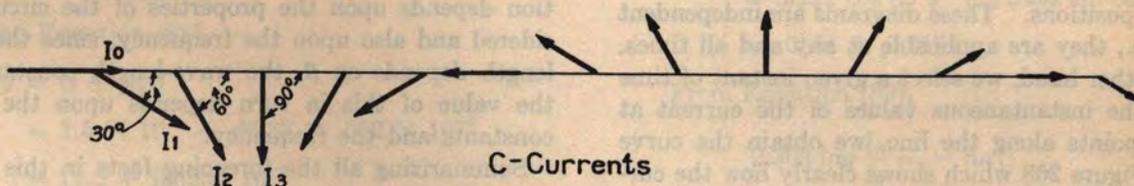
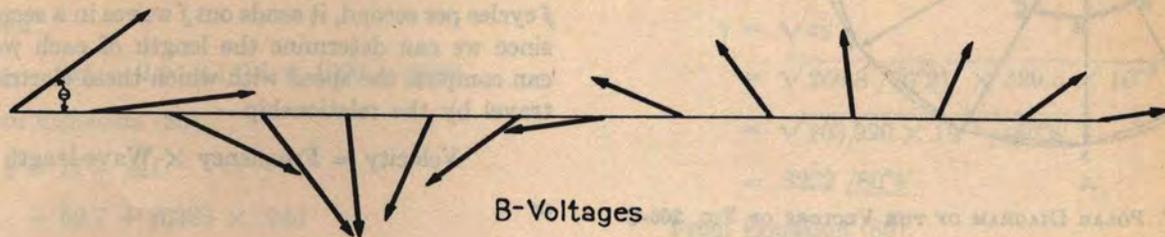
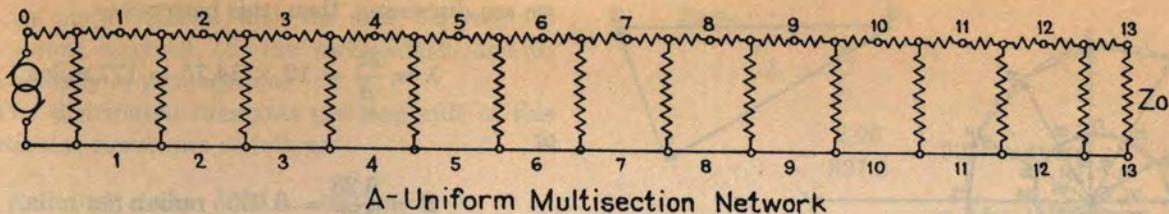


FIG. 266. TRANSMITTED CURRENTS AND VOLTAGES AT JUNCTIONS OF A MULTISECTION UNIFORM NETWORK

our sections of such a length that, for a frequency of 1000 cycles, the time lag between voltages can be represented by 30 degrees per section on the **time-voltage** diagram; if we simulate by each section fourteen and three-quarters miles of 104 open wire circuit, we will obtain such a relationship. In order to make the story complete, we will also assume the reduction in voltage magnitude due to resistance and leakage loss to be 0.895 per section. If we assume the original voltage  $E_0$  to be 10 volts, the voltage at the end of the first section,  $E_1$ , will be 8.95 volts, lagging  $30^\circ$  behind  $E_0$ .  $E_2$ , at the end of the second section, will be  $0.895 \times 8.95$  or 8.01 volts, lagging  $30^\circ$  behind  $E_1$  or  $60^\circ$  behind  $E_0$ . If we represent the voltages at various points by vectors, we will obtain a system of vectors as shown in Figure 266-B, where the multisection network is shown as Figure 266-A and the voltage acting at each junction is directly below.

Since the ratio of current to voltage is constant,\* it follows that the chart representing currents will have the same form, with each vector proportional and removed by an angle  $\theta$  from the corresponding voltage vector, where  $\theta$  is the angle of the characteristic impedance  $Z_0$ . Thus we may treat a similar figure such as 266-C as a "distance-current diagram" where the vectors,  $I_0, I_1, I_2$ , etc., show the magnitude and relative phase of the currents at the network junctions. If now

we refer all the current vectors to a common reference point, we will obtain a broken curve such as that of Figure 267-A, which shows graphically how the currents at various points are related. In this figure the vector  $I_0 = G-0$  is the current entering the first section and  $I_1 = G-1$ , the current leaving that section. Then the vector 1-0 must be the current that passes through the shunt in the first section, because the sum of the current through the shunt and the current going ahead gives 1-0 as the resultant of the vector diagram. This is perhaps more clearly illustrated by Figure 267-B. For the same reason 2-1 will be the current passing through the second shunt, etc.

We may, therefore, conceive of the total entering current as the resultant of a number of component currents which flow from the generator through the various shunt paths and back to the generator, each component of a different magnitude and phase. The effect of these components can be observed, since at certain junctions the line current is flowing in the opposite direction to that taken by the entering current; at other points there is a  $90^\circ$  phase difference between the two; and at still other points there is no phase difference. In other words, the current vector may be considered as moving about  $G$ , rotating through  $30^\circ$  for every section traversed and diminishing in value about 10% in each section.

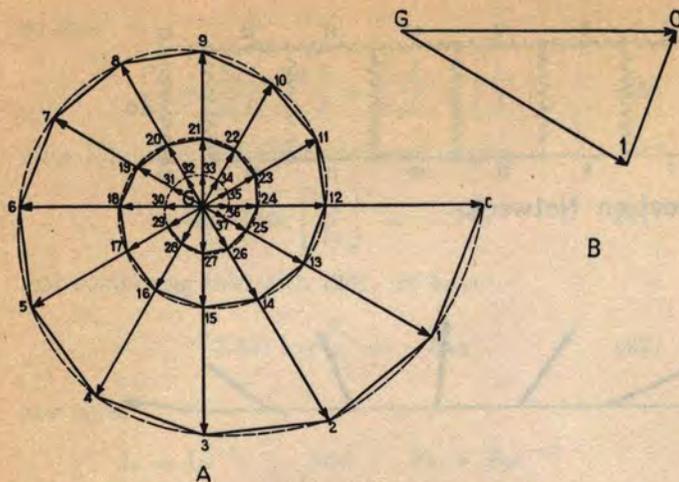


FIG. 267. POLAR DIAGRAM OF THE VECTORS OF FIG. 266-C

Figures 266 and 267 show the effective values of the current at certain points along the line and their relative phase positions. These diagrams are independent of time, i.e., they are applicable at any and all times. If on the other hand, we select a given instant of time and plot the instantaneous values of the current at the same points along the line, we obtain the curve shown in Figure 268 which shows clearly how the current reverses in direction as it passes through the various sections. A little study of this curve suggests that it is related to the sine curve, and such is actually the case. Due to the decay of current from section to section, the sine wave is somewhat distorted, but if the decay were eliminated, the curve would be a pure sine wave. A comparison of the method used to obtain Figure 268 with the method of deriving the sine curve will show this clearly.

Figure 268 shows graphically both ways in which the line has affected the propagation of the wave. The decrease in the height of each successive cycle illustrates the attenuation of the current. The fact that we have a succession of cycles plotted against distance instead of time also shows how there has been established by the medium a definite speed of propagation. For the particular frequency there is a definite length, viz., 12, which as expressed here is the number of sections for one complete wave. We may call this wavelength  $\lambda$ , and indicate a definite relation between  $\lambda$  and  $\beta$  as follows:

$$\lambda = \frac{2\pi}{\beta} \quad (94)$$

since  $\beta$  is a constant for the line at a given frequency and is a measure of the amount of phase shift per section, and when obtained from Equation (86) or (88), is on the basis of radian measure. In other words, there are 360 degrees or  $2\pi$  radians in one wavelength (or one cycle)  $\lambda$ . For the particular network

we are discussing, then, this becomes—

$$\lambda = \frac{2\pi}{\beta} = 12 \times 14.75 = 177 \text{ miles}$$

or

$$\beta = \frac{6.28}{177} = 0.0355 \text{ radian per mile.}$$

Now, we know that if an E.M.F. has a frequency of  $f$  cycles per second, it sends out  $f$  waves in a second, and since we can determine the length of each wave, we can compute the speed with which these electric waves travel by the relationship—

$$\text{Velocity} = \text{Frequency} \times \text{Wave-length}$$

or

$$W = f\lambda \quad (95)$$

It can thus be seen that the speed of electric propagation depends upon the properties of the circuit considered and also upon the frequency, since the wavelength depends on  $\beta$ , the wave-length constant, and the value of this in turn depends upon the circuit constants and the frequency.

Summarizing all the foregoing facts in this article, we have for our transmission line—

- Both current and voltage are retarded.
- Both current and voltage are attenuated.
- The amount of attenuation and the amount of "slowing down" are determined by the physical properties of the circuit and by the frequency of the applied voltage.

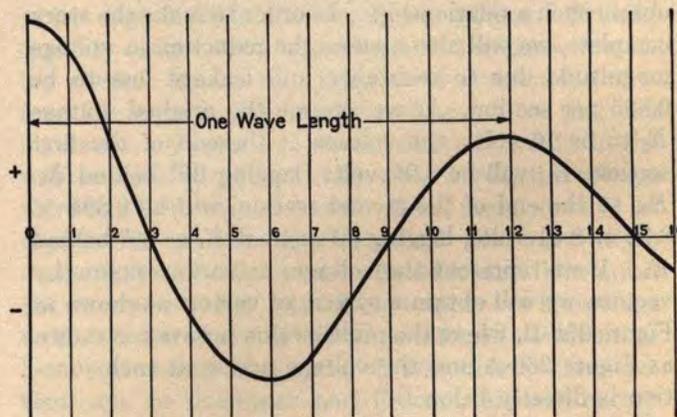


FIG. 268. STANDING WAVE OF CURRENT ALONG UNIFORM TRANSMISSION LINE

**Example:** Assuming a 50-mile, 19-gage H-44 side circuit terminated in its characteristic impedance and with an input power at the sending end of 10 milliwatts, calculate at 1000 cycles per second (1) the characteristic impedance, (2) the magnitude of the received current at the distant end and

(3) its phase relation with the sent current, (4) the power received, (5) the wave-length, and (6) the velocity of propagation.

The distributed constants per loop mile of this particular circuit are as follows:

$$R = 89.7 \text{ ohms} \quad C = .062 \text{ mf.}$$

$$L = .040 \text{ henry} \quad G = 1.5 \text{ m. mhos.}$$

**Solution:**

$$\omega = 2\pi f = 2 \times 3.1416 \times 1000 = 6283$$

From Equation (80),

$$\begin{aligned} z &= R + j\omega L \\ &= 89.7 + j6283 \times .040 \\ &= 89.7 + j251.3 \\ &= 266.8 / 70^\circ 21' \end{aligned}$$

From Equation (81),

$$\begin{aligned} y &= G + j\omega C \\ &= 1.5 \times 10^{-6} + j6283 \times .062 \times 10^{-6} \\ &= (1.5 + j389.5) 10^{-6} \\ &= 389.5 \times 10^{-6} / 89^\circ 47' \end{aligned}$$

From Equation (82),

$$\begin{aligned} Z_0 &= \sqrt{\frac{z}{y}} = \sqrt{\frac{266.8 / 70^\circ 21'}{389.5 \times 10^{-6} / 89^\circ 47'}} \\ &= \sqrt{684,980 / -19^\circ 26'} \\ &= 827.5 / -9^\circ 43' \quad \text{Ans. (1).} \end{aligned}$$

The input power,  $P_0$ , is

$$P_0 = E_0 I_0 \cos \theta$$

Substituting

$$I_0 = E_0 / Z_0 \text{ in the above,}$$

$$P_0 = \frac{E_0^2 \cos \theta}{Z_0}$$

or

$$E_0^2 = \frac{P_0 Z_0}{\cos \theta}$$

(Note: When  $Z_0$  is a pure resistance,  $\theta$  is zero and its cosine is one. Therefore, when  $\theta$  is small in value it may, for all practical purposes, be disregarded.)

$$E_0^2 = \frac{.010 \times 827.5}{.9856} = 8.396$$

$$E = 2.90 \text{ volts}$$

Then

$$\begin{aligned} I_0 &= \frac{E_0}{Z_0} \\ &= \frac{2.90}{827.5} = .0035 \text{ ampere} \end{aligned}$$

or 3.5 milliamperes

From Equation (85)

$$\begin{aligned} \gamma &= \sqrt{zy} \\ &= \sqrt{266.8 / 70^\circ 21' \times 389.5 \times 10^{-6} / 89^\circ 47'} \\ &= \sqrt{103,920 \times 10^{-6} / 160^\circ 8'} \\ &= .3222 / 80^\circ 4' \end{aligned}$$

From Equation (86),

$$\begin{aligned} \gamma &= \alpha + j\beta \\ &= .3222 \cos 80^\circ 4' + j.3222 \sin 80^\circ 4' \\ &= .0556 + j.3174 \end{aligned}$$

From Equation (89),

$$2.303 \log \frac{I_n}{I_0} = -n\alpha$$

or

$$2.303 \log \frac{I_0}{I_n} = n\alpha$$

$$\log \frac{3.50}{I_n} = \frac{50 \times .0556}{2.303} = 1.207$$

$$\frac{3.50}{I_n} = 16.11$$

$$I_n = \frac{3.50}{16.11} = .22 \text{ milliampere} \quad \text{Ans. (2)}$$

Phase shift per mile =  $\beta$

$$\beta = .3174 \text{ radian or } 18.2^\circ$$

Total phase shift for 50-mile circuit is

$$50 \times 18.2 = 910^\circ \quad \text{Ans. (3).}$$

Then  $I_n = .22 / 910^\circ$  milliampere

From Equation (89),

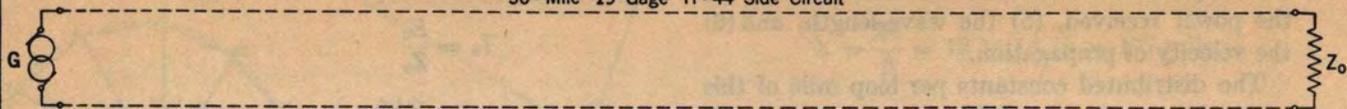
$$2.303 \log \frac{E_0}{E_n} = n\alpha$$

$$\log \frac{2.90}{E_n} = \frac{50 \times .0556}{2.303} = 1.207$$

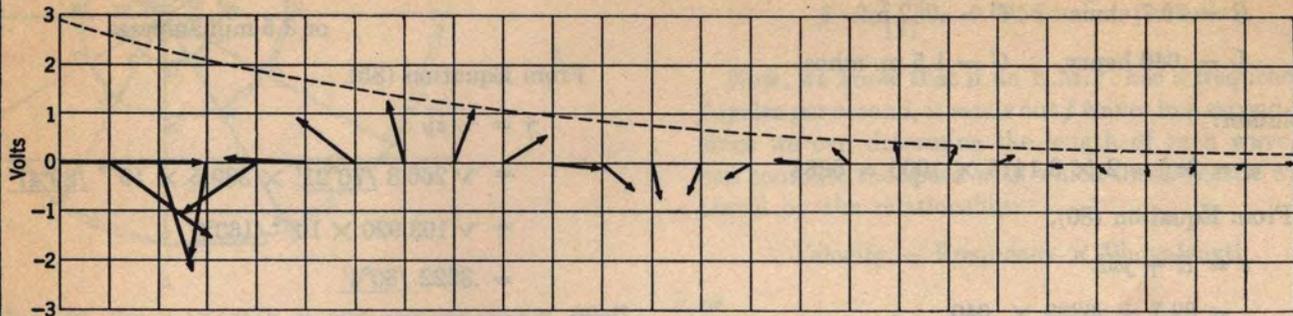
$$\frac{2.90}{E_n} = 16.11$$

$$E_n = \frac{2.90}{16.11} = .18 \text{ volt}$$

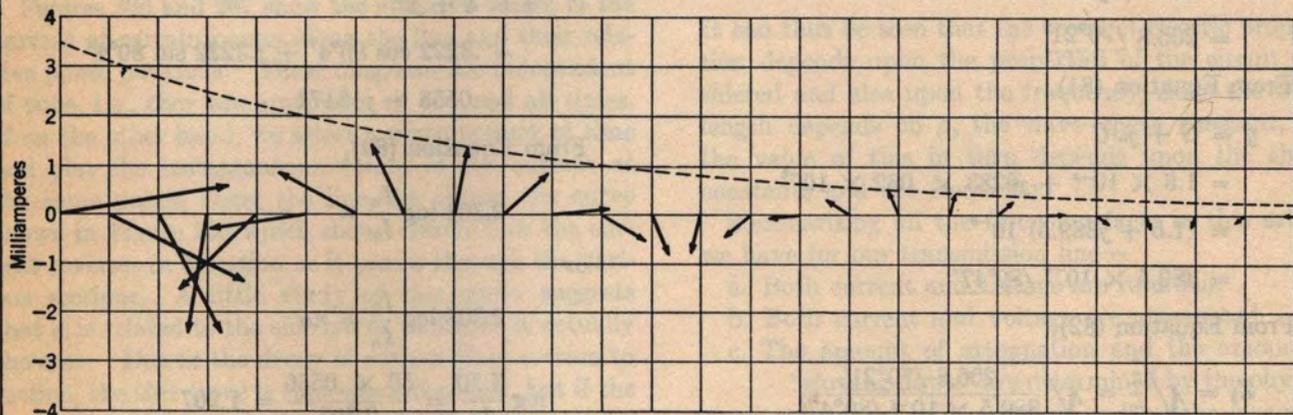
50 - Mile 19 Gage H - 44 Side Circuit



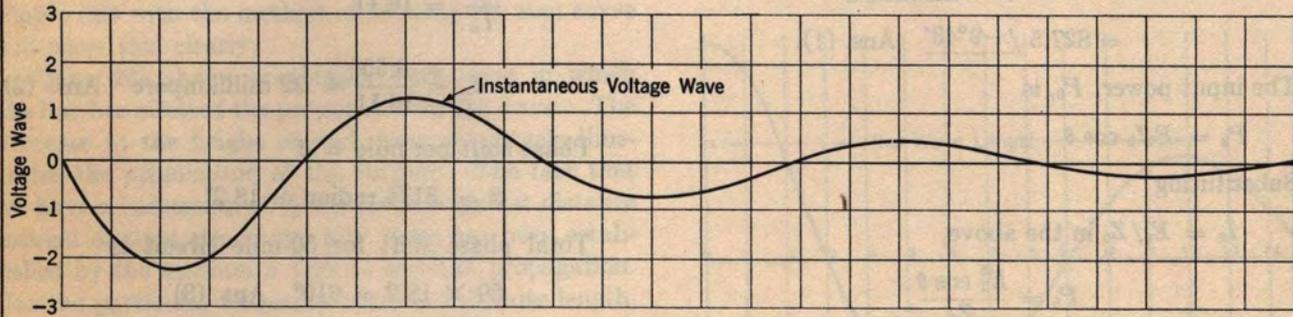
A



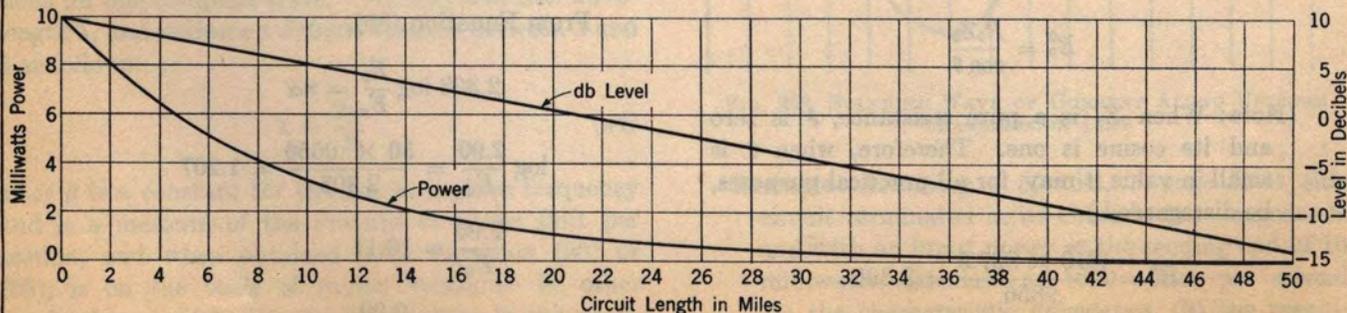
B



C



D



E

FIGURE 269

From Equation (93),

$$\begin{aligned} P_n &= E_n I_n \cos \theta \\ &= .18 \times .00022 \times .9856 \\ &= .000039 \text{ watt} \\ &\text{or } .039 \text{ milliwatt.} \end{aligned}$$

Ans. (4)

From Equation (94),

$$\begin{aligned} \lambda &= \frac{2\pi}{\beta} \\ &= \frac{6.283}{.3174} = 19.79 \text{ miles.} \end{aligned}$$

Ans. (5)

From Equation (95),

$$\begin{aligned} W &= f\lambda \\ &= 1000 \times 19.79 \\ &= 19,790 \text{ miles per second.} \end{aligned}$$

Ans. (6)

The conditions along the line are graphically illustrated by Figure 269. The ordinates of the dashed curves in B and C represent the magnitudes of effective voltage and current at all points throughout the length of the circuit. The voltage and current vectors represent both magnitude and phase relation at the end of each 2-mile section. The standing voltage wave on the line is shown in D. This curve represents the instantaneous voltage values at all points on the line, i.e., the sine of the voltage vectors in B. The power at all points along the line is shown by E. As the power is proportional to the product of  $EI \cos \theta$ , it decreases faster than either the effective voltage or current values illustrated by the dashed curves in B and C.

## 127. Reflection and Transition Loss

We have noted that it is a characteristic of wave motion that in passing from one medium to another, a certain amount of the energy propagated by the wave is lost. For instance, light waves striking a pane of glass, water, or some denser medium are in part transmitted and in part reflected. The amount of energy reflected depends on the physical properties of the media through which the wave passes, the greater the dissimilarity, the greater the reflection. We may consider that such reflection is due to the different velocities with which the dissimilar media propagate energy, so that at the junction some interaction takes place, the result of which produces reflection, i.e., a change in the amount of energy propagated.

In transmitting electric waves, this reflection phe-

nomenon is frequently met with, and it causes a "reflection loss". The amount of loss can actually be computed or measured, and if we take the case of two unequal impedances  $Z_1$  and  $Z_2$ , the ratio of power received to the power that would be received on a smooth circuit ( $Z_1 = Z_2$ ) is given by—

$$\frac{P_2}{P_1} = \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2} \times \frac{\cos \theta_2}{\cos \theta_1} \quad (96)$$

where  $Z_1$  is one vector impedance with an angle  $\theta_1$ ,  $Z_2$  is the other vector impedance with an angle  $\theta_2$ , and the direction of propagation is from  $Z_1$  to  $Z_2$ .

In practical telephone work the factor  $\frac{\cos \theta_2}{\cos \theta_1}$  is usually neglected, due to the fact that in the ordinary connection, substation to substation, this factor cancels out when considering the total reflection loss on the circuit. If it is remembered that reflection loss is a reduction in energy which is met with in all forms of wave propagation, a clearer conception of this phenomenon is obtained.

There is another so-called loss met with in transmission work which is known as "transition loss". In the preceding chapter we learned that if the load resistance was not equal to the resistance of the system to which it was connected, the power received by the load would not be a maximum. Similarly, in A.C. circuits, certain conditions must be met in order that the load may receive maximum power. Briefly stated, these conditions are that **the resistance of the load must equal the resistance of the generator and the reactance of the load must be of the same magnitude as the reactance of the generator but of opposite sign.** When these conditions prevail the two reactance components will cancel one another so that the circuit will behave as a D.C. circuit. It naturally follows that the resistance components must follow the D.C. law given in Chapter XIX. The transition loss, so-called, is in effect a comparison of the power that is received by a load under any given circuit conditions with the power that could be received if conditions permitted the maximum transfer of power. Usually this reduction in power is given in the form of a ratio in the same way that reflection loss is given by a ratio. If we designate by  $P_2$  the power that is actually received by the load and by  $P_1$  the maximum power that could be received, the ratio of the two is given by—

$$\frac{P_2}{P_1} = \frac{4R_1 R_2}{(Z_1 + Z_2)^2} \quad (97)$$

Transition loss is not a true physical loss, nor is it a measure of the efficiency of the circuit; it is merely an indication of what percentage of the maximum power possible of utilization is being utilized.

## 128. Units for the Measurement of Transmission Losses and Gains

As in dealing with any other quantity, we require some unit of measurement when dealing with the energy losses due to attenuation in the transmission of human speech, or in the transmission of any alternating current from a sending device to a receiving device over a long line or through complicated circuits. Without some such unit we would be handicapped in giving any scientific expression to the grade of telephone transmission under various conditions. It would be natural for us to say that sufficient energy had been transmitted from the speaking station to the listening station for the listener to hear distinctly every spoken word, or to say that the sound coming from the receiver at the receiving station was so faint as not to be intelligible, but this would be a crude method of comparison. For the same reason that we need some adopted standard as a unit of length, such as the foot or the meter to measure distance, we require some standard for the measurement of transmission loss or transmission gain in telephone work.

For many years the unit used for this purpose was the "standard cable mile". This represented the loss due to one mile of an old type of standard 19-gage cable, having a resistance of 88 ohms per loop mile and a capacity of .054 mf. per mile. In this cable the series inductance and the shunt leakage were negligible, while the bridged capacity of .054 mf. was appreciable. It therefore attenuated the various frequencies that make up the band for telephone transmission unequally, attenuating the higher frequencies more than the lower frequencies. To illustrate, the attenuation constant  $\alpha$  was equal to .109 for 800-cycle frequency and .122 for 1000-cycle frequency, etc.

This meant that the percentage reduction in power caused by inserting a mile of standard cable between a sending and receiving element was different for different frequencies. Under these conditions, to say that a telephone circuit had an equivalent of a certain number of miles of standard cable was largely meaningless unless the frequency at which the equivalent was computed or measured was stated at the same time. This rather confusing situation led to the dropping of the mile of standard cable altogether as a unit of measurement and the substitution of an arbitrarily selected unit not differing greatly in magnitude from the standard cable mile through the voice range, but having exactly the same significance at any and all frequencies. That is to say, the new unit, called the decibel (abbreviated "db"), represents always a fixed percentage reduction in power no matter what frequency is involved. Its magnitude may perhaps be best grasped by remembering that in a circuit equating to ten db the output power will always be one-tenth of the input

TABLE XI  
RELATION BETWEEN DECIBELS AND POWER RATIOS  
FOR GAINS AND LOSSES

DECIBELS	APPROXIMATE POWER RATIO		
	For Losses		For Gains
	Fractional	Decimal	Decimal
1	4/5	.8	1.25
2	2/3	.63	1.6
3	1/2	.5	2.0
4	2/5	.4	2.5
5	1/3	.32	3.2
6	1/4	.25	4.0
7	1/5	.2	5.0
8	1/6	.16	6.0
9	1/8	.125	8.0
10	1/10	.1	10.0
20	1/100	.01	100.0
30	1/1000	.001	1000.0

power. Mathematically, the power ratio for one db may be expressed as—

$$\frac{P_1}{P_0} = 10^{-1} \quad (98)$$

where  $P_1$  is input power and  $P_0$  is output power. This corresponds to a current ratio of  $10^{-0.5}$  and to an attenuation constant value of  $\alpha = .115$ . Table XI showing the power ratios for several values of decibels will aid in forming a clear conception of the magnitude of the unit.

For any given power ratio the number of db corresponding can be determined by the following simple formula—

$$\text{No. of db} = N = 10 \log \frac{P_1}{P_0} \quad (99)$$

or, if the current ratio rather than the power ratio is known—

$$N = 20 \log \frac{I_1}{I_0} \quad (100)$$

or from Equation (77)—

$$\begin{aligned} N &= 20 \log \frac{I_1}{I_0} = 20 \log e^\alpha = 20 \times \alpha \times \log e \\ &= 20 \times .434 \times \alpha = 8.68\alpha \end{aligned} \quad (101)$$

Although in the above we have been considering the decibel in connection with measurements of "loss" or attenuation, it is equally useful in the measurement of "gain" such as that given by a telephone repeater. A telephone repeater would be said to have a gain of so many db if the circuit in connection with which it were used was effectively shortened or had its net attenuation reduced to that extent.